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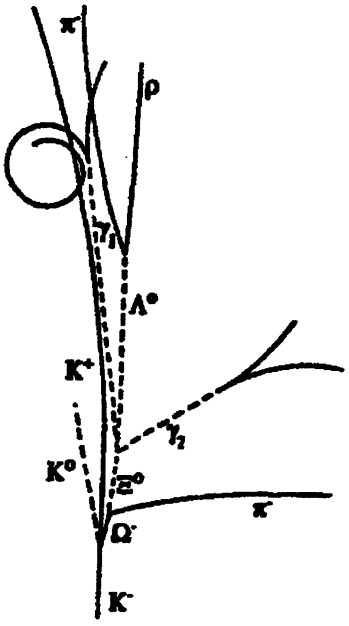

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TIME SCALES APPROACH TO MINKOWSKI PLANE MOTION

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Abstract

In this study our aim was investigating a new approximation for the one-parameter kinematics of Minkowski plane by using time scale method. By using the formulas of Minkowski motion we investigated the delta-derivative equations of the one-parameter motion on $\mathbb{T}$. Then we gave the velocities and their union rule. Finally by using the delta-derivative, we got some results for the instantaneous rotation pole and pole curves of the motion on the time scale.

Keywords: Time scale, Minkowski plane, motion, kinematic.
AMS: 34B10, 34B15

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1 Introduction

The calculus on time scales was initiated by B. Aulbach and S. Hilger [1,7] in order to create a theory that can unify discrete and continuous analysis. Some preliminary definitions and theorems about delta derivative can also be found in the book [5].

In this study, our main aim is to apply Minkowski one-parameter planar motion's properties in references [2,3,5,6] on time scale. We first take two Minkowski planes on time scale plane $(\mathbb{T}, \mathbb{R})$, then investigate the modelling of Minkowski one-parameter planar motion on time scale. By using some time scale hyperbolic function's properties of [6], we find delta-derivative equations of the motion. Finally we get some theorems and results about pole curves.

2 Preliminaries

A time scale is an arbitrary nonempty closed subset of the real numbers.

Definition 2.1 *Let $\mathbb{T}$ be any time scale. The forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$ is defined by $\sigma(t) := \inf\{s \epsilon \mathbb{T} : s > t\}$ and the backward jump operator $\rho : \mathbb{T} \to \mathbb{T}$ is defined by $\rho(t) := sup\{s \epsilon \mathbb{T} : s < t\}$, for $t \in \mathbb{T}$.*

Theorem 2.1 *Assume $f, g : \mathbb{T} \to \mathbb{R}$ are differentiable at $t \in \mathbb{T}^k$. Then:*
(i) The sum $f + g : \mathbb{T} \to \mathbb{R}$ is differentiable at t with

$$(f+g)^{\Delta}(t) = f^{\Delta}(t) + g^{\Delta}(t).$$

(ii) For any constant $\alpha, \alpha f : \mathbb{T} \to \mathbb{R}$ is differentiable at t with

$$(\alpha f)^{\Delta}(t) = \alpha f^{\Delta}(t).$$

(iii) The product $fg : \mathbb{T} \to \mathbb{R}$ is differentiable at t with

$$(fg)^{\Delta}(t) = f^{\Delta}(t)g(t) + f(\sigma(t))g^{\Delta}(t) = f(t)g^{\Delta}(t) + f^{\Delta}(t)g(\sigma(t)).$$

(iv) If $f(t)f(\sigma(t)) \neq 0$ then $\frac{1}{f}$ is differentiable at t with

$$(\frac{1}{f})^{\Delta}(t) = -\frac{f^{\Delta}(t)}{f(t)f(\sigma(t))}.$$

(v) If $g(t)g(\sigma(t)) \neq 0$, then $\frac{f}{g}$ is differentiable at t with

$$(\frac{f}{g})^{\Delta}(t) = \frac{f^{\Delta}(t)g(t) - f(t)g^{\Delta}(t)}{g(t)g(\sigma(t))}.$$

3 The Minkowski Plane Motion on Time Scale

Assume that $\mathbb{T}$ is an any time scale. Let L^2 and L'^2 be moving and fixed Minkowski planes on the time scale, and $\{0, \overrightarrow{\ell}_1, \overrightarrow{\ell}_2\}$ and $\{0', \overrightarrow{\ell}'_1, \overrightarrow{\ell}'_2\}$ be their orthonormal frames, respectively. Here, for simplify, we will denote the Minkowski planes L^2 and L'^2 as L and L'. Then we say that L: $\{0, \overrightarrow{\ell}_1, \overrightarrow{\ell}_2\}$ moves with respect to L': $\{0', \overrightarrow{\ell}'_1, \overrightarrow{\ell}'_2\}$, $\overrightarrow{\ell}_i$, (i=1,2) are the time-like vectors for every t parameter on the time scale.

Definition 3.1 *By taking a translation vector on Minkowski plane*

$$\overrightarrow{00'} = \overrightarrow{u} = u_1\overrightarrow{\ell_1} + u_2\overrightarrow{\ell_2} \tag{3.1}$$

and a rotation angle φ between the vectors $\overrightarrow{\ell}'_1$ and $\overrightarrow{\ell}_1$ (or the Minkowski planes L and L'), the motion is called as Minkowski one-parameter planar motion on time scale and denoted by $L/_{\mathbb{L}}L'$ for a Minkowski planar motion of L relative to L'.

Definition 3.2 *The delta derivative of the rotation angle φ, $\varphi^{\Delta}_{\mathbb{L}}(t) = \frac{d\varphi}{\Delta t}$ is called a delta-angular velocity of the Minkowski motion $L/_{\mathbb{L}}L'$ on time scale.*

The time-like vectors $\overrightarrow{\ell}'_1$ and $\overrightarrow{\ell}_1$ can be written as following:

$$\overrightarrow{\ell_1} = \overrightarrow{\ell'_1}[\cosh_p(t, t_0)o\varphi(t)] + \overrightarrow{\ell'_2}[\sinh_p(t, t_0)o\varphi(t)] \tag{3.2}$$

$$\overrightarrow{\ell_2} = \overrightarrow{\ell'_1}[\sinh_p(t, t_0)o\varphi(t)] + \overrightarrow{\ell'_2}[\cosh_p(t, t_0)o\varphi(t)] \tag{3.3}$$

with the restriction $\varphi^{\Delta}_{\mathbb{L}}(t) \neq 0$, $p = 1$, and $t_0 = 0$. Here assume the functions

$$u_1 = u_1(t), \qquad u_2 = u_2(t), \qquad \varphi = \varphi(t)$$

are $\Delta-$differentiable functions and the parameter t is defined as $t_0 \leq t \leq t_1$ on $\mathbb{T}$ time scale. Let $\overrightarrow{x} = (x_1, x_2)$ and $\overrightarrow{x}' = (x_1', x_2')$ be the position vectors of a point X on the Minkowski planes, with respect to the moving and fixed orthonormal frames L and L', respectively. Then

$$\overrightarrow{x} = \overrightarrow{OX} = \overrightarrow{\ell}_1 x_1 + \overrightarrow{\ell}_2 x_2 \quad and \quad \overrightarrow{x}' = \overrightarrow{O'X} = \overrightarrow{\ell}'_1 x_1' + \overrightarrow{\ell}'_2 x_2', \tag{3.4}$$

is obvious. Here it is easy to see following equation

$$\overrightarrow{O'X} = \overrightarrow{O'O} + \overrightarrow{OX} = -\overrightarrow{OO'} + \overrightarrow{OX},$$

then we get

$$\overrightarrow{x'} = -\overrightarrow{u} + \overrightarrow{x} = \overrightarrow{\ell}_1(-u_1 + x_1) + \overrightarrow{\ell}_2(-u_2 + x_2). \tag{3.5}$$

When the coordinates of the point $X \in L$ is taken independent from the time $t \in \mathbb{T}$, the point X becomes a fixed point on the Minkowski plane L or $X \in L$. On the contrary if the coordinates x_1', x_2' of the same X point are constant numbers, X stays fixed on the plane L' or X belongs to the L' plane. It is easy to see that we can obtain a different formula on time scale as following:

Remark 3.1 *Relation between the point* $X = (x_1, x_2)$ *and* $X^* = (x_1^\sigma, x_2)$ *can be defined as*

$$\overrightarrow{X} = \overrightarrow{X}^* + \overrightarrow{X^*X},$$

here $t \in \mathbb{T}$ *and* x_1^σ *is a forward jump function of* x_1.

4 The Delta-Derivative Equations of the Motion on Time Scale

Assume that $\varphi : \mathbb{T} \to \mathbb{R}$ is strictly increasing and $\tilde{\mathbb{T}} = \varphi(\mathbb{T})$ is a time scale. Let $\sinh_p, \cosh_p : \tilde{\mathbb{T}} \to \mathbb{R}$, and $\varphi_{\mathbb{L}}^{\Delta}(t)$, $sinh_p^{\tilde{\Delta}}(t, t_0)$, $cosh_p^{\tilde{\Delta}}(t, t_0)$ exist

for $t \in \mathbb{T}^k$. By taking Δ-derivative of Eqs. 3.2 and 3.3

$$\begin{aligned}
\overrightarrow{\ell_1^{\Delta}} &= [\overrightarrow{\ell_1}'(\cosh_p(t,t_0)o\varphi(t)) + \overrightarrow{\ell_2}'(\sinh_p(t,t_0)o\varphi(t))]^{\Delta} \\
&= \overrightarrow{\ell_1}'[\cosh_p(t,t_0)o\varphi(t)]^{\Delta} + \overrightarrow{\ell_2}'[\sinh_p(t,t_0)o\varphi(t)]^{\Delta} \\
&= \overrightarrow{\ell_1}'[\cosh_p^{\tilde{\Delta}}(t,t_0)o\varphi(t)]\varphi_{\mathbb{L}}^{\Delta}(t) + \overrightarrow{\ell_2}'[\sinh_p^{\tilde{\Delta}}(t,t_0)o\varphi(t)]\varphi_{\mathbb{L}}^{\Delta}(t) \\
&= \overrightarrow{\ell_1}'[p(t)\sinh_p(t,t_0)o\varphi(t)]\varphi_{\mathbb{L}}^{\Delta}(t) + \overrightarrow{\ell_2}'[p(t)\cosh_p(t,t_0)o\varphi(t)]\varphi_{\mathbb{L}}^{\Delta}(t) \\
&= p(t)\varphi_{\mathbb{L}}^{\Delta}(t)[\overrightarrow{\ell_1}'\sinh_p(t,t_0)o\varphi(t) + \overrightarrow{\ell_2}'\cosh_p(t,t_0)o\varphi(t)]
\end{aligned}$$

$$\begin{aligned}
\overrightarrow{\ell_2^{\Delta}} &= [\overrightarrow{\ell_1}'(\sinh_p(t,t_0)o\varphi(t)) + \overrightarrow{\ell_2}'(\cosh_p(t,t_0)o\varphi(t))]^{\Delta} \\
&= \overrightarrow{\ell_1}'[\sinh_p(t,t_0)o\varphi(t)]^{\Delta} + \overrightarrow{\ell_2}'[\cosh_p(t,t_0)o\varphi(t)]^{\Delta} \\
&= \overrightarrow{\ell_1}'[\sinh_p^{\tilde{\Delta}}(t,t_0)o\varphi(t)]\varphi_{\mathbb{L}}^{\Delta}(t) + \overrightarrow{\ell_2}'[\cosh_p^{\tilde{\Delta}}(t,t_0)o\varphi(t)]\varphi_{\mathbb{L}}^{\Delta}(t) \\
&= \overrightarrow{\ell_1}'[p(t)\cosh_p(t,t_0)o\varphi(t)]\varphi_{\mathbb{L}}^{\Delta}(t) + \overrightarrow{\ell_2}'[p(t)\sinh_p(t,t_0)o\varphi(t)]\varphi_{\mathbb{L}}^{\Delta}(t) \\
&= p(t)\varphi_{\mathbb{L}}^{\Delta}(t)[\overrightarrow{\ell_1}'\cosh_p(t,t_0)o\varphi(t) + \overrightarrow{\ell_2}'\sinh_p(t,t_0)o\varphi(t)]
\end{aligned}$$

are obtained for $p = 1, t = 0$. By using Eq. 3.2 and Eq. 3.3 we can get following equations

$$\overrightarrow{\ell_1}^{\Delta} = p(t)\varphi_{\mathbb{L}}^{\Delta}(t)\overrightarrow{\ell_2}, \quad \overrightarrow{\ell_2}^{\Delta} = p(t)\varphi_{\mathbb{L}}^{\Delta}(t)\overrightarrow{\ell_1}. \tag{4.1}$$

By the same method, using Eq. 4.1, we can find the Δ-derivative of Eq. 3.1 as

$$\begin{aligned}
\overrightarrow{u}_{\mathbb{L}}^{\Delta}(t) &= [u_1(t)\overrightarrow{\ell_1}(t) + u_2(t)\overrightarrow{\ell_2}(t)]^{\Delta} \\
&= [u_1(t)\overrightarrow{\ell_1}(t)]^{\Delta} + [u_2(t)\overrightarrow{\ell_2}(t)]^{\Delta} \\
&= u_1^{\Delta}(t)\overrightarrow{\ell_1}(t) + u_1^{\sigma}\overrightarrow{\ell_1}^{\Delta}(t) + u_2^{\Delta}(t)\overrightarrow{\ell_2}(t) + u_2^{\sigma}\overrightarrow{\ell_2}^{\Delta}(t) \\
&= u_1^{\Delta}(t)\overrightarrow{\ell_1}(t) + u_1^{\sigma}p(t)\varphi_{\mathbb{L}}^{\Delta}(t)\overrightarrow{\ell_2}(t) + u_2^{\Delta}(t)\overrightarrow{\ell_2}(t) + u_2^{\sigma}p(t)\varphi_{\mathbb{L}}^{\Delta}(t)\overrightarrow{\ell_1}(t)
\end{aligned}$$

So we get following equation:

$$\overrightarrow{u}_{\mathbb{L}}^{\Delta}(t) = \overrightarrow{\ell_1}[u_1^{\Delta}(t) + u_2^{\sigma}p(t)\varphi_{\mathbb{L}}^{\Delta}(t)] + \overrightarrow{\ell_2}[u_2^{\Delta}(t) + u_1^{\sigma}p(t)\varphi_{\mathbb{L}}^{\Delta}(t)]. \tag{4.2}$$

The Eq.4.1 and Eq.4.2 are Δ-derivative equations of the Lorentz motion $L/_{\mathbb{L}}L'$ on time scale. It is obvious that the planar motion with one parameter is included in these equations.

5 The Velocities and Their Union Rule of The Minkowski Motion on Time Scale

While the Minkowski planar motion of L plane is moving relative to L' plane, the point X move on the moving plane L by $t \in \mathbb{T}$. Thus here, two Minkowski motion will be investigated to get the union rule of all velocities on the time scale.

Definition 5.1 *A velocity vector of the point X with respect to the Minkowski plane L, i.e. a vectoral velocity, which is belong to a curve (X) of the point X on the plane L, is called Δ-relative velocity vector of the point X on time scale and is denoted by $\overrightarrow{v}_{\mathbb{L}\Delta r}$.*

From the Eq.3.4

$$\overrightarrow{x}_{\mathbb{L}} = \overrightarrow{\ell_1} x_1 + \overrightarrow{\ell_2} x_2$$

for the velocity vector, by taking $\Delta-$derivative of $\overrightarrow{x}_{\mathbb{L}}$, by using Δ derivative, as taking $\overrightarrow{\ell_1}$ and $\overrightarrow{\ell_2}$ fixed, we can obtain following equation as $\Delta-$relative velocity vector

$$\overrightarrow{v}_{\mathbb{L}\Delta r} = \overrightarrow{x}_{\mathbb{L}}{}^{\Delta} = \overrightarrow{\ell_1} x_1^{\Delta} + \overrightarrow{\ell_2} x_2^{\Delta}. \tag{5.1}$$

Definition 5.2 *A velocity vector of the point X with respect to the plane L' which is belong to a curve (X') of the point X' on L' is called Minkowski Δ-absolute velocity vector of the point X on time scale and is denoted by $\overrightarrow{v}_{\mathbb{L}\Delta a}$.*

Definition 5.3 *On the planar motion $L/_{\mathbb{L}}L'$, while the point X is fixed on the moving plane E(i.e.$\overrightarrow{v}_{\mathbb{L}\Delta r} = \overrightarrow{0}$), a velocity vector of the point X is called the Δ-dragging velocity vector of this point on time scale and is denoted by $\overrightarrow{v}_{\mathbb{L}\Delta f}$.*

Theorem 5.1 *A Δ-absolute velocity vector of the Minkowski plane motion is equal to adding Δ-relative velocity vector and Δ-dragging velocity vector on the motion $L/_{\mathbb{L}}L'$, i.e.*

$$\overrightarrow{v}_{\mathbb{L}\Delta a} = \overrightarrow{v}_{\mathbb{L}\Delta f} + \overrightarrow{v}_{\mathbb{L}\Delta r}. \tag{5.2}$$

Proof. By using Eq.4.1, Eq.4.2 and Eq.5.1, the Δ-derivative of Eq. 3.5 with respect to parameter t is

$$\begin{aligned}\overrightarrow{v}_{\mathbb{L}\Delta a} &= \overrightarrow{x}'^{\Delta}_{\mathbb{L}} = -u^{\Delta}_{\mathbb{L}} + x^{\Delta}_{\mathbb{L}} \\ &= -\{\overrightarrow{\ell_1}[u_1^{\Delta}(t) + p(t)u_2^{\sigma}u^{\Delta}(t)] + \overrightarrow{\ell_2}[u_2^{\Delta}(t) + p(t)u_1^{\sigma}u^{\Delta}(t)]\} \\ &\quad +\{x_1^{\Delta}\overrightarrow{\ell_1} + x_1^{\sigma}\overrightarrow{\ell_1}^{\Delta}(t) + x_2^{\Delta}\overrightarrow{\ell_2} + x_2^{\sigma}\overrightarrow{\ell_2}^{\Delta}(t)\}\end{aligned}$$

and from here, we can obtain following equation as the $\Delta-$ absolute velocity vector of the Minkowski plane motion:

$$\begin{aligned}\overrightarrow{v}_{\mathbb{L}\Delta a} &= \overrightarrow{\ell_1}[-u_1^{\Delta}(t) - p(t)u_2^{\sigma}\varphi_{\mathbb{L}}^{\Delta}(t) + p(t)\varphi_{\mathbb{L}}^{\Delta}(t)x_2^{\sigma}] \\ &\quad +\overrightarrow{\ell_2}[-u_2^{\Delta}(t) - p(t)u_1^{\sigma}\varphi_{\mathbb{L}}^{\Delta}(t) + p(t)x_1^{\sigma}\varphi_{\mathbb{L}}^{\Delta}] + \overrightarrow{v}_{\mathbb{L}\Delta r}.\end{aligned}$$

Here, because of the Definition 5.3 (i.e. $\overrightarrow{v}_{\mathbb{L}\Delta r} = \overrightarrow{0}$), the vector

$$\begin{aligned}\overrightarrow{v}_{\mathbb{L}\Delta f} &= \overrightarrow{\ell_1}[-u_1^{\Delta} + (-u_2^{\sigma} + x_2^{\sigma})p(t)\varphi_{\mathbb{L}}^{\Delta}(t)] \\ &\quad +\overrightarrow{\ell_2}[-u_2^{\Delta} + (-u_1^{\sigma} + x_1^{\sigma})p(t)\varphi_{\mathbb{L}}^{\Delta}]\end{aligned}$$

is the Δ-dragging velocity vector of the Minkowski plane motion on time scale.

If the point X is fixed on the plane L, i.e. $\overrightarrow{v}_{\mathbb{L}\Delta r} = \overrightarrow{0}$, from the Eq.5.2, the Δ-absolute velocity vector of the Minkowski plane motion is equal to the Δ-dragging velocity vector of the Minkowski plane motion:

$$\overrightarrow{v}_{\mathbb{L}\Delta a} = \overrightarrow{v}_{\mathbb{L}\Delta f}$$

6 The Minkowski Instantaneous Rotation Pole and Pole Curves on Time Scale

Here, we will investigate the points at which the Δ-dragging velocity of the Minkowski plane motion is zero for any instant $t \in \mathbb{T}$ on the motion $L/_{\mathbb{L}}L'$ on time scale.

Theorem 6.1 *There is only one point at which the Δ-dragging velocity vector is zero for any instant $t \in \mathbb{T}$, i.e., which is fixed on the both planes L and L', with the restriction $\varphi_{\mathbb{L}}^{\Delta} \neq 0$ on the motion $L/_{\mathbb{L}}L'$.*

Proof. The points at which the Δ-dragging velocity is zero for any instant $t \in \mathbb{T}$ have to stay fixed for not only the plane L, but also for the plane L' on the motion $L/_{\mathbb{L}}L'$. By taking $\overrightarrow{v}_{\mathbb{L}\Delta f} = \overrightarrow{0}$, we get the components of the Δ-dragging vector becomes zero. So we have:

$$-u_1^{\Delta} + (-u_2^{\sigma} + x_2^{\sigma})p(t)\varphi_{\mathbb{L}}^{\Delta}(t) = 0 \tag{6.1}$$
$$-u_2^{\Delta} + (-u_1^{\sigma} + x_1^{\sigma})p(t)\varphi_{\mathbb{L}}^{\Delta}(t) = 0 \tag{6.2}$$

Since $\varphi^{\Delta} \neq 0$ and if $p \neq 0$, from Eq.6.1 and Eq.6.2, we obtain following relations:

$$p_1^{\sigma} = u_1^{\sigma} + \frac{u_2^{\Delta}}{p(t)\varphi_{\mathbb{L}}^{\Delta}(t)} \tag{6.3}$$

$$p_2^{\sigma} = u_2^{\sigma} + \frac{u_1^{\Delta}}{p(t)\varphi_{\mathbb{L}}^{\Delta}(t)}. \tag{6.4}$$

Definition 6.1 *The point $P(p_1^{\sigma}, p_2^{\sigma})$ which is corresponding to the position vector $\overrightarrow{OP} = \overrightarrow{P} = \overrightarrow{\ell_1}p_1^{\sigma} + \overrightarrow{\ell_2}p_2^{\sigma}$ is called the Minkowski forward pole or instantaneous rotation pole or instantaneous rotation center for $t \in \mathbb{T}$ on the motion $L/_{\mathbb{L}}L'$, on time scale.*

Also if we calculate equations from Eq.6.3 and Eq.6.4

$$u_1^{\Delta} = [p_2^{\sigma} - u_2^{\sigma}]p(t)\varphi_{\mathbb{L}}^{\Delta}(t)$$
$$u_2^{\Delta} = [p_1^{\sigma} - u_1^{\sigma}]p(t)\varphi_{\mathbb{L}}^{\Delta}(t).$$

and substitute them in Eq. 5.3, we get following:

$$\begin{aligned}\overrightarrow{v}_{\mathbb{L}\Delta f} &= \overrightarrow{\ell}_1[-u_1^{\Delta} + (-u_2^{\sigma} + x_2^{\sigma})p(t)\varphi_{\mathbb{L}}^{\Delta}(t)] + \overrightarrow{\ell}_2[-u_2^{\Delta} - u_1^{\sigma} + x_1^{\sigma})p(t)\varphi_{\mathbb{L}}^{\Delta}(t)] \\ &= \overrightarrow{\ell}_1[-(p_2^{\sigma} - u_2^{\sigma})p(t)\varphi_{\mathbb{L}}^{\Delta}(t) + (-u_2^{\sigma} + x_2^{\sigma})p(t)\varphi_{\mathbb{L}}^{\Delta}(t)] \\ &\quad + \overrightarrow{\ell}_2[-(p_1^{\sigma} - u_1^{\sigma})p(t)\varphi_{\mathbb{L}}^{\Delta}(t) + (-u_1^{\sigma} + x_1^{\sigma})p(t)\varphi_{\mathbb{L}}^{\Delta}(t)]\end{aligned}$$

Then we obtain the equation of the Δ-dragging velocity vector of the Minkowski plane motion as:

$$\overrightarrow{v}_{\mathbb{L}\Delta f} = \{\overrightarrow{\ell}_1[x_2^{\sigma} - p_2^{\sigma}] + \overrightarrow{\ell}_2[x_1^{\sigma} - p_1^{\sigma}]\}\varphi_{\mathbb{L}}^{\Delta}(t)p \tag{6.5}$$

This relation denotes an expression of the Δ-dragging velocity vector of the point X. From Eq.6.5 we can obtain following results:

7 Conclusion

Corollary 7.1 *Since the Minkowski scalar product of the vector*

$$\overrightarrow{PX}^{\sigma} = \overrightarrow{\ell}_1(x_1^{\sigma} - p_1^{\sigma}) + \overrightarrow{\ell}_2(x_2^{\sigma} - p_2^{\sigma}) \tag{7.1}$$

and the vector $\overrightarrow{v}_{\mathbb{L}\Delta f}$ is zero, these vectors are perpendicular because of their Minkowski dot product.

Proof.

$$\begin{aligned}
\langle \overrightarrow{P}X^{\sigma}, \overrightarrow{v}_{\mathbb{L}\Delta f}\rangle_{\mathbb{L}} =& \langle \overrightarrow{\ell}_1(x_1^{\sigma} - p_1^{\sigma}) + \overrightarrow{\ell}_2(x_2^{\sigma} - p_2^{\sigma}),\\
& \overrightarrow{\ell}_1(x_2^{\sigma} - p_2^{\sigma}) + \overrightarrow{\ell}_2(x_1^{\sigma} - p_1^{\sigma})\varphi_{\mathbb{L}}^{\Delta}(t)p\rangle_{\mathbb{L}}\\
=& \varphi_{\mathbb{L}}^{\Delta}(t)p\langle \overrightarrow{\ell}_1(x_1^{\sigma} - p_1^{\sigma}) + \overrightarrow{\ell}_2(x_2^{\sigma} - p_2^{\sigma}),\\
& \overrightarrow{\ell}_1(x_2^{\sigma} - p_2^{\sigma}) + \overrightarrow{\ell}_2(x_1^{\sigma} - p_1^{\sigma})\rangle_{\mathbb{L}}\\
=& \varphi_{\mathbb{L}}^{\Delta}(t)p\{(x_1^{\sigma} - p_1^{\sigma})(x_2^{\sigma} - p_2^{\sigma}) - (x_2^{\sigma} - p_2^{\sigma})(x_1^{\sigma} - p_1^{\sigma})\}\\
=& 0.
\end{aligned}$$

Theorem 7.1 *For the Minkowski motion $L/_{\mathbb{L}}L'$ on time scale, the points X of the moving Minkowski plane L draw trajectories on the fixed the plane L' which their normals pass from instantaneous rotation pole P.*

Corollary 7.2 *From Eq. 6.5 and Eq. 7.1 the length of the vector $\overrightarrow{v}_{\mathbb{L}\Delta f}$ can be calculated as following:*

$$\|\overrightarrow{v}_{\mathbb{L}\Delta f}\| = \sqrt{(x_1^{\sigma} - p_1^{\sigma})^2 - (x_2^{\sigma} - p_2^{\sigma})^2}p(t)\varphi_{\mathbb{L}}^{\Delta} = \|\overrightarrow{PX}^{\sigma}\|\,|\varphi_{\mathbb{L}}^{\Delta}|p(t)\,,$$

here $\|\overrightarrow{PX^{\sigma}}\|$ is used for the length of $\overrightarrow{PX^{\sigma}}$ on Lorentz plane. So we get the following relation

$$\|\overrightarrow{v}_{\mathbb{L}\Delta f}\| = \|\overrightarrow{P}X^{\sigma}\|\,|\varphi_{\mathbb{L}}^{\Delta}|p(t), \tag{7.2}$$

from the Eq. 7.2 we get the following theorem:

Theorem 7.2 *Every X^{σ} jump operator of X of the moving plane L is doing (instantaneous) rotation movement with a P-centered, $\varphi_{\mathbb{L}}^{\Delta}$-angular velocity and p factor on instant t.*

Theorem 7.3 *An one-parameter Minkowski motion is consist of rotation with $\varphi_{\mathbb{L}}^{\Delta}$ angular velocity and p factor around the instantaneous rotation pole P of the moving plane E on t instant, i.e. the plane L rotates with the angle $\varphi_{\mathbb{L}}^{\Delta}$ and the factor p around the point P on the time element Δt.*

Definition 7.1 *The locus of the point P is usually a curve on the moving plane L, this curve is called the moving pole curve (P), and the locus of the point P is called the fixed pole curve (P') on the plane L' of the motion $L/_{\mathbb{L}}L'$ on time scale.*

Theorem 7.4 *The velocity vectors of the forward instantaneous rotation pole P which draws pole curves on the moving and fixed planes is the same vector at each instant t.*

From this theorem, at each instant t, both pole curves (P) and (P') is tangent on the instantaneous rotation pole P; and the rotation pole (P) in the Δt time interval have equal arc elements on both pole curves:

$$d_{\mathbb{L}}s' = \|\overrightarrow{v}_{\mathbb{L}\Delta a}\| \cdot \Delta t = \|\overrightarrow{v}_{\mathbb{L}\Delta r}\| \cdot \Delta t = d_{\mathbb{L}}s$$

On the other hand, two curve become continuously tangent to each other, and if there are equal arc elements between the mutual contact point, we call that these two curves roll onto each other without sliding. Thus, the following theorem can be given as:

Theorem 7.5 *On the Minkowski one-parameter planar motion H_I the moving pole curve (P) of the plane L rolls onto fixed pole curve (P') of the plane L' without sliding on time scale.*

Corollary 7.3 *Without being depended on time, a Minkowski motion $L/_{\mathbb{L}}L'$ on time scale occurs by rolling, without sliding, the curve (P) of L onto the curve (P') of L'.*

We can get following properties with some special case of Theorem 7.5:
1) One of both curves, for example if the curve (P) is constrict to a P point, this case become same as the other (P') curve. Thus the motion $L/_{\mathbb{L}}L'$ on time scale is consist of a rotation around the fixed point P.

2) If $\varphi_{\mathbb{L}}^{\Delta}$ becomes zero for definite value of $t \in \mathbb{T}$, according to Eq.5.3 all $X(x_1, x_2)$ points of moving plane L have the same dragging velocity:

$$\overrightarrow{v}_{\mathbb{L}\Delta f} = -(\overrightarrow{\ell}_1 u_1^{\Delta} + \overrightarrow{\ell}_2 u_2^{\Delta}).$$

Thus the limit $\varphi_{\mathbb{L}}^{\Delta} \to 0$ means that (from Eq.6.3 and Eq.6.4), the rotation pole P goes to infinite, and at this time both pole curves (P) and (P') have infinitely distance point(and asymptote).
3) If $\varphi_{\mathbb{L}}^{\Delta} = 0$ for all value of $t \in \mathbb{T}$ continuously, i.e. if $\varphi = constant$, we get a translating motion. Thus the motion can not occur by rolling pole curves.

Remark 7.1 *We can use nabla derivative for this paper and take similar results.*

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ISO-GALOIS FIELD AND ISOPERMUTATION GROUP

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Abstract

The concept of 'Isotopy' as formulated by Ruggero Maria Santilli [1, 2, 3] plays a vital role in the development of Iso mathematics. Santilli defined iso-fields of characteristic zero. In this paper we investigate Iso-Galois fields [4] which are essentially of non-zero characteristic. Isopermutation group [13] defines a clear cut distinction between automorphic groups and isotopic groups.

keywords:- Isounit, Isofield, Galois field and Iso-Galois field, Isotopically isomorphic realization of a group, Isopermutation group.

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1 Introduction

During a talk at the conference *Differential Geometric Methods in Mathematical Physics* held in Clausthal, Germany, in 1980, Ruggero Maria Santilli submitted new numbers based on certain axiom preserving generalization of the multiplication, today known as *isotopic numbers* or *isonumbers*[1] in short. This generalization induced the so-called isotopies of the conventional multiplication with consequential generalization of the multiplicative unit, where the Greek word "isotopy" implied the meaning "same topology". Subsequently, Rugero Maria Santilli submitted a new conjugation, under the name *isoduality* which yields an additional class of numbers, today known as *isodual isonumbers* [1].
The discovery of isonumbers was made with the specific need of quantitative representation of the transition from :
a) *exterior dynamical problem*, i.e., particles moving in the homogeneous and isotropic vacuum (empty space) with consequential local-differential and potential-cannonical equations of motion, to
b) *interior dynamical problem*, i.e. extended and therefore deformable particles moving within an inhomogeneous and anisotropic physical medium, with consequential equations of motion of the most general known nonlinear, nonlocal-integral and nonpotential-noncanonical type.
We appraise the readers with some basic ideas of Santilli's isofields and some related results.

1.1 Santilli's Isofield

Definition 1.1. *Given a field F with elements $\alpha, \beta, \gamma, \ldots$, sum $\alpha + \beta$, multiplication $\alpha\beta$, and respective units 0 and 1, "Santilli's isofields" are rings of elements $\hat{\alpha} = \alpha\hat{1}$ where α are elements of F and $\hat{1} = \hat{T}^{-1}$ is a positive-definite $n \times n$ matrix generally outside F equipped with the same sum $\hat{\alpha} + \hat{\beta}$ of F with related additive unit $\hat{0} = 0$ and a new multiplication $\hat{\alpha} * \hat{\beta} = \hat{\alpha}\hat{T}\hat{\beta}$, under which $\hat{1} = \hat{T}^{-1}$ is the new left and right unit of F in which case $\hat{F}$ satisfies all axioms of a field.*

The 'isofields' $\hat{F} = \hat{F}(\hat{\alpha}, +, \hat{\times})$ are given by elements $\hat{\alpha}, \hat{\beta}, \hat{\gamma} \ldots$ characterized by one-to-one and invertible maps $\alpha \to \hat{\alpha}$ of the original element

$\alpha \in F$ equipped with two operations $(+, \hat{\times})$, the conventional addition $+$ of F and a new multiplication $\hat{\times}$ called "isomultiplication" with corresponding conventional additive unit 0 and a generalized multiplicative unit $\hat{1}$, called "multiplicative isounit" under which all the axioms of the original field F are preserved.

If the conventional field is chosen to be alternative under the operation of conventional multiplication then the resulting isofield is also isoalternative under isomultiplication.
If the given algebraic structure is a noncommutative division ring (e.g. ring of quaternions) then the resulting isoalgebraic structure is also noncommutative under the isomultiplication.

This new algebraic structure has revolutionized contemporary mathematics and found its applications in so far unexplored (unexplained) and unknown territories of quantum mechanics and quantum chemistry [5, 6].
The resulting new theory of numbers has become the basis of recent studies of **nonlinear-nonlocal, non-Hamiltonian systems** in nuclear particle and statistical physics. Santilli iso-numbers, which are the mathematical basis of Hadronic Mechanics, are also introduced and reviewed in [10].

Santilli's Isofields [1], [5] are of two types, **isofield of first kind**; wherein the isounit does not belong to the original field, and **isofield of second kind**; wherein the isounit belongs to the original field. The elements of the isofield are called as **isonumbers**. This leads to number of new terms and parallel developments of conventional mathematics.

We mention two important propositions by Santilli [2] which classify isofields of second kind and isofields of first kind respectively.

Proposition 1.1. *The necessary and sufficient condition for the lifting (where the multiplication is lifted but elements are not)* $F(a, +, \times) \longrightarrow \hat{F}(a, +, \hat{\times}), \hat{\times} = \times\hat{T}\times, \hat{1} = \hat{T}^{-1}$ *to be an isotopy (that is for* $\hat{F}$ *to verify all axioms of the original field F) is that* $\hat{T}$ *is a non-null element of the original field* F.

e.g. We can start with the field of real numbers $\Re$ and construct an isotopic field $\hat{\Re}$ with a new multiplicative identity as $\hat{1} = \frac{1}{2}$ where $\hat{T} = 2$ as the isounit element. So, if a and $b \in \Re$ then the isoproduct of a and b will be $a\hat{\times}b = a.2.b = 2ab$.

Proposition 1.2. *The lifting (where both the multiplication and the elements are lifted)* $F(a,+,\times) \longrightarrow \hat{F}(\hat{a},+,\hat{\times}), \hat{a} = a \times \hat{1}, \hat{\times} = \times\hat{T}\times, \hat{1} = \hat{T}^{-1}$ *constitutes an isotopy even when the multiplicative isounit* $\hat{1}$ *is not an element of the original field.*

The following three theorems [12] directly follow from the definition of isofield.

Proposition 1.3. *If* $F(a,+,\times)$ *is a field and* $\hat{F}(a,+,\hat{\times})$ *is the corresponding isofield such that the isounit* $\hat{I} \in F$ *then* $F(a,+,\times) \cong \hat{F}(a,+,\hat{\times})$.

Clearly, the map $x \longmapsto \hat{x}$ is an isomorphism.

Proposition 1.4. *If* $F(a,+,\times)$ *is a field and* $\hat{F}(\hat{a},+,\hat{\times})$ *is the corresponding isofield such that the isounit* $\hat{I} \notin F$ *then* $F(a,+,\times)$ *is isotopic to* $\hat{F}(\hat{a},+,\hat{\times})$.

Proposition 1.5. *Isofield corresponding to a non-commutative field is isotopic to the original field.*

The noncommutative ring of Quaternions is an example of this type.

2 Galois fields

Definition 2.1. *A field having finite number of elements is called a Galois field.*

2.1 Isofields $\hat{F}_m$ and $\hat{F}_{em}$

In view of Proposition 1.1 we can easily derive the following new Proposition.

Proposition 2.1. *The necessary and sufficient condition for the lifting (where the multiplication and elements both are lifted)* $F(a,+,\times) \longrightarrow \hat{F}(\hat{a},+,\hat{\times})$, $\hat{\times} = \times\hat{T}\times, \hat{1} = \hat{T}^{-1}$ *to be an isotopy (that is for* $\hat{F}$ *to verify all axioms of the original field F) is that* $\hat{T}$ *is a non-null element of the original field* F.

Proof. The field axioms can be readily verified for the resulting isofield. □

Thus for a given field $F(a,+,\times)$ with a non-null element $\hat{T} \in F$ we can construct two types of isofields isotopic to F as;

1. The isofield $\hat{F}_m = \hat{F}(a,+,\hat{\times})$ isotopic to F in which only the multiplication is lifted but the elements are not lifted.

2. The isofield $\hat{F}_{em} = \hat{F}(\hat{a},+,\hat{\times})$ isotopic to F in which the elements and the multiplication both are lifted.

Given a field F the following two Propositions give the relationship between the $\hat{F}_m$ and $\hat{F}_{em}$.

Proposition 2.2. *If* F *is a field,* $a, b \in F$ *and,* x *and* y *are the corresponding isotopic (or conventional) products of* a *and* b *in* $\hat{F}_m$ *and* $\hat{F}_{em}$ *respectively (i.e.* $x \in \hat{F}_m$ *and* $y \in \hat{F}_{em}$*), then* $x = y\hat{T}^2$ *where* $\hat{T}$ *is the isotopic element for* $\hat{F}_m$ *and* $\hat{F}_{em}$.

Proof. x is in $\hat{F}_m$ implies $x = a\hat{T}b = (\frac{a}{\hat{T}}.\hat{T}.\frac{b}{\hat{T}}).\hat{T}^2 = (\hat{a}.\hat{\times}.\hat{b})\hat{T}^2 = y\hat{T}^2$, where $a, b \in F$. □

Proposition 2.3. *If* F *is a field such that* $\hat{T}$ *is a non-null element of* F*, then lifting of multiplication in* F *by* $\hat{T}$ *without lifting the elements of* F *identically is same as simultaneous isotopic lifting of elements in* F *and multiplication by* $\hat{T}^{-1}$.

Proof. Let $a, b \in F$. Lifting product in F by $\hat{T}$ we get $a\hat{\times}b = a\hat{T}b = (a\hat{T})\frac{1}{\hat{T}}(b\hat{T})$. □

e.g. consider the the field of reals R. Let the isotopic element be 3. Then lifting of the product of 5 and 7 by 3 yields $5\hat{\times}7 = 5.3.7 = 105$. Also, lifting the numbers 5 and 7 by 3 and product by $\frac{1}{3}$ yields the same result; $(5.3).\frac{1}{3}.(7.3) = 5.3.7 = 105$.

Corollary 2.1. *Every isofield of $\hat{F}_m$ with isoelement $\hat{T} \in F$ is an isofield $\hat{F}_{em}$ with isoelement $\frac{1}{\hat{T}}$.*

Finite fields were first introduced by Galois in 1830 in his proof of the un-solvability of the general quintic equation. Hence finite fields are also called as Galois fields. When Cayley invented matrices a few decades later, it was natural to investigate groups of matrices over finite fields. In fact, the groups of matrices over the finite fields have become the most important class of groups. Finite fields have vast applications in computer science, coding theory, information theory, and cryptography.

Thus, Galois fields are finite fields. Finite fields are of nonzero characteristic. Every finite field is of prime-power order, and for every power of a prime there is a unique Galois field of this order.
Santilli's isofields are defined for the fields of characteristic zero, and hence for infinite fields.
Our main purpose is to apply Santilli's ideas to the fields of non-zero characteristic and seek for further development in this direction.
In this paper we answer the open problems posed in [12],

- Can we construct finite isofields of first kind ?
- Can we construct finite isofields of second kind ?

in the affirmative.

3 Iso-Galois fields

Definition 3.1. *If F is an iso-field and F is finite, then F is called an Iso-Galois field.*

Definition 3.2. *Let F be a Galois field. If G is an Iso-Galois field of F wherein the prescribed multiplicative identity is from the field F itself, then the Iso-Galois field G is called an* ***Iso-Galois field of second kind.***

Definition 3.3. *Let F be a Galois field. If G is an Iso-Galois field of F wherein the prescribed multiplicative identity is not from the field F, then the Iso-Galois field G is called an* ***Iso-Galois field of first kind.***

If G is an Iso-Galois field constructed from the field F then we call the field F as the **trivial iso-field**.

4 Iso-Galois fields of second kind

Consider a Galois field F_8 as a set of following matrices of integers modulo 2.

$$(0) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, (1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, (2) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, (3) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix},$$
$$(4) = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, (5) = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}, (6) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, (7) = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

For construction of an isofield it will be sufficient to consider the following multiplication table of non-zero elements of F_8.

$\cdot$	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1)	1	2	3	4	5	6	7
(2)	2	3	4	5	6	7	1
(3)	3	4	5	6	7	1	2
(4)	4	5	6	7	1	2	3
(5)	5	6	7	1	2	3	4
(6)	6	7	1	2	3	4	5
(7)	7	1	2	3	4	5	6

Let us choose the iso-element $\hat{T} = (4) \equiv 4 \in F_8$ in the above composition table. Then the iso-unit element is $\hat{1} = \frac{1}{\hat{T}} = \frac{1}{4} = 4^{-1} = 5$. We now construct the corresponding composition table for isomultiplication in $\hat{F}_m$

using the fact that the isomultiplication $\hat{\times}$ is defined as $a\hat{\times}b = a\hat{T}b$.
e.g. $6\hat{\times}7 = 6\cdot 4\cdot 7 = 1$ using above composition table. Thus the corresponding isomultiplication table (or iso-composition table) for isonumbers (here same as original numbers) will be;

$\hat{\times}$	$\widehat{(1)}$	$\widehat{(2)}$	$\widehat{(3)}$	$\widehat{(4)}$	$\widehat{(5)}$	$\widehat{(6)}$	$\widehat{(7)}$
$\widehat{(1)}$	4	5	6	7	1	2	3
$\widehat{(2)}$	5	6	7	1	2	3	4
$\widehat{(3)}$	6	7	1	2	3	4	5
$\widehat{(4)}$	7	1	2	3	4	5	6
$\widehat{(5)}$	1	2	3	4	5	6	7
$\widehat{(6)}$	2	3	4	5	6	7	1
$\widehat{(7)}$	3	4	5	6	7	1	2

Similarly, we can construct the corresponding composition table for isomultiplication in $\hat{F}_{em}$ using the fact that the isomultiplication $\hat{\times}$ is defined as $\hat{a}\hat{\times}\hat{b} = \frac{a}{\hat{T}}\hat{T}\frac{b}{\hat{T}}$.
We know that for $a \in F_8$, $\hat{a} = a \cdot \frac{1}{\hat{T}} = a \cdot 5$. Hence first we construct the corresponding isonumbers. Thus, $\hat{2} = 2 \cdot \frac{1}{4} = 2 \cdot 5 = 6$. Similarly, $\hat{3} = 7$, $\hat{4} = 1$, $\hat{3} = 7$, $\hat{4} = 1$, $\hat{5} = 2$, $\hat{6} = 3$ and $\hat{7} = 4$.
Also the isoproduct of the isonumbers $\hat{6}$ and $\hat{7}$ is $\hat{6}\hat{\times}\hat{7} = 3 \cdot 4 \cdot 4 = 2$ using above composition table. Thus the corresponding isomultiplication table (or iso-composition table) for isonumbers (not same as the original numbers) will be;

$\hat{\times}$	$\widehat{(1)}$	$\widehat{(2)}$	$\widehat{(3)}$	$\widehat{(4)}$	$\widehat{(5)}$	$\widehat{(6)}$	$\widehat{(7)}$
$\widehat{(1)}$	5	6	7	1	2	3	4
$\widehat{(2)}$	6	7	1	2	3	4	5
$\widehat{(3)}$	7	1	2	3	4	5	6
$\widehat{(4)}$	1	2	3	4	5	6	7
$\widehat{(5)}$	2	3	4	5	6	7	1
$\widehat{(6)}$	3	4	5	6	7	1	2
$\widehat{(7)}$	4	5	6	7	1	2	3

Remark. *1. The function $f : F_8 \to F_8$ defined by $f(x) = 5x$ is not an isomorphism.*
2. However, the isofunction $\hat{f} : x \to \hat{x}$ is an isomorphism.

We generalize these observations in the following Theorem.

Theorem 4.1. *If F is a Galois field such that $\hat{F}$ is an Iso-Galois field of second kind, where $\hat{T} \in F$ is an isoelement and $\hat{x} = \hat{T}^{-1}x$, $x \in F$ then the function $f : F \to F$ defined by $x \mapsto \hat{T}^{-1}x$ is not an isomorphism but is an isotopism, whereas the isofunction $\hat{f} : F \to \hat{F}$ is an isomorphism.*

Proof. It is easy to verify that the function f is a translation from F to F and hence is not an isomorphism. It is an isotopism because we have prescribed a new identity and the result follows from [12].
If we consider the isofunction $\hat{f} : F \to \hat{F}$ then for $x, y \in F$, $\hat{f}(x.y) = \hat{T}^{-1}x.y$ whereas $\hat{f}(x)\hat{\times}\hat{f}(y) = \hat{x}\hat{\times}\hat{y} = \hat{T}^{-1}x.\hat{T}.\hat{T}^{-1}y = \hat{T}^{-1}x.y$. Thus $\hat{f}(x.y) = \hat{f}(x)\hat{\times}\hat{f}(y)$. □

Theorem 4.2. *If F is a Galois field of order p^m and n is the number of involutions in F then there exist $p^m - n - 1$ number of distinct Iso-Galois fields of 2nd kind of F.*

Proof. The multiplicative group of F will obviously contain $p^m - 1$ number of non-zero elements. Every involution and its inverse will obviously give rise to the same Iso-Galois field of second kind. Therefore the total number of distinct Iso-Galois fields would be $p^m - n - 1$. □

5 Iso-Galois fields of First kind

Consider a Galois field F of order 16 represented by the polynomials $a + bx + cx^2 + dx^3$, a, b, c and d are integers modulo 2. The polynomials are generated by the powers of x using the rule $x^4 = 1 + x$.
The elements of the field are; $(0) = (0,0,0,0)$, $(1) = (1,0,0,0)$, $(2) = (0,1,0,0)$, $(3) = (0,0,1,0)$, $(4) = (0,0,0,1)$, $(5) = (1,1,0,0)$, $(6) = (0,1,1,0)$, $(7) = (0,0,1,1)$, $(8) = (1,1,0,1)$, $(9) = (1,0,1,0)$, $(10) = (0,1,0,1)$, $(11) = (1,1,1,0)$, $(12) = (0,1,1,1)$, $(13) = (1,1,1,1)$, $(14) = (1,0,1,1)$, $(15) = (1,0,0,1)$ with the following composition table for multiplication;

·	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1
3	3	4	5	6	7	8	9	10	11	12	13	14	15	1	2
4	4	5	6	7	8	9	10	11	12	13	14	15	1	2	3
5	5	6	7	8	9	10	11	12	13	14	15	1	2	3	4
6	6	7	8	9	10	11	12	13	14	15	1	2	3	4	5
7	7	8	9	10	11	12	13	14	15	1	2	3	4	5	6
8	8	9	10	11	12	13	14	15	1	2	3	4	5	6	7
9	9	10	11	12	13	14	15	1	2	3	4	5	6	7	8
10	10	11	12	13	14	15	1	2	3	4	5	6	7	8	9
11	11	12	13	14	15	1	2	3	4	5	6	7	8	9	10
12	12	13	14	15	1	2	3	4	5	6	7	8	9	10	11
13	13	14	15	1	2	3	4	5	6	7	8	9	10	11	12
14	14	15	1	2	3	4	5	6	7	8	9	10	11	12	13
15	15	1	2	3	4	5	6	7	8	9	10	11	12	13	14

The set $F_1 = \{0, 1, 6, 11\}$ forms a subfield of F. We consider an element $\hat{T} = 2$ such that $\hat{T}$ does not belong to F_1 and form an Iso-Galois field of F_1. The isounit $\hat{1} = \frac{1}{\hat{T}} = \hat{T}^{-1} = 2^{-1} = 15$. The numbers of F_1 are converted to following isonumbers as $\hat{1} = 15$, $\hat{6} = 6.15 = 5$ and $\hat{11} = 11.15 = 10$. Thus the isofield is $\hat{F}_1 = \{0, 15, 5, 10\}$ with the following composition table for isomultiplication.

$\hat{\times}$	15	5	10
15	15	5	10
5	5	10	15
10	10	15	5

e.g the isoproduct of 5 and 10 is given by $5.\hat{T}.10 = 5.2.10 = 15$.
Similarly, if $\hat{T} = 7$ then $\hat{1} = \frac{1}{\hat{T}} = 7^{-1} = 10$. The numbers of F_1 are converted to following isonumbers as $\hat{1} = 10$, $\hat{6} = 6.10 = 15$ and $\hat{11} = 11.10 = 5$. Thus the isofield is $\hat{F}_1 = \{0, 10, 15, 5\}$ with the following composition table for isomultiplication.

$\hat{\times}$	10	15	5
10	10	15	5
15	15	5	10
5	5	10	15

Theorem 5.1. *If F is a Galois field of order p^n and F_1 is a subfield of F of order p^m such that $F \setminus F_1$ has r number of involutions, then there exist $p^n - p^m - r$ number of distinct Iso-Galois fields of first kind of F_1.*

Proof. F_1 is a subfield of F implies $m|n$. Let $n = m + r$. Then
$o(F \setminus F_1) = p^n - p^m = p^{m+r} - p^m = p^m(p^r - 1) \ldots (1)$.
Case I : If $p = 2$ then $(1) \Rightarrow o(F \setminus F_1)$ is even.
Case II ; If $p \neq 2$ then $p^r - 1$ is even and again $o(F \setminus F_1)$ is even.
Also, if $x \in F \setminus F_1$ then x^{-1} also belongs to $F \setminus F_1$. Thus, each $x \in F \setminus F_1$ gives rise to one isofield of first kind of F_1. Now, if x is an involution then $x = x^{-1}$ and hence x and x^{-1} will give rise to same isofield. If there are r number of involutions in $F \setminus F_1$ then the number of elements which give rise to distinct isofields will be $p^n - p^m - r$. Hence the result. □

6 Isotopically Isomorphic Realization

Santilli's isofields have multiplicative group which is isotopically isomorphic realization (when the isounit is from the field itself) or isotopically isomorphic representation (when the isounit is from outside the field) of the multiplicative group of the original field. For a given group G if we consider all permutations of the group elements, then there exist permutations which

are the right regular representations, and permutations which are the left regular representations of G. In this case, each permutation is actually an isogroup with respect to the induced isounit and isomultiplication. Thus the permutation group of a group G always contains the group of automorphisms of G and the group of isopermutations (left and right) of G. Hence representation (or realization) of a group G as (left and/or right) isopermutation group can have extensive applications in cryptography, coding theory and biological sciences wherein the codes and the keys are to be continuously changing for the strongest security.

Isotopism of the fields [1, 2, 3] defined by R.M. Santilli induces isotopism of the groups in the following manner.

Definition 6.1. *If G is a group then the permutation f_l (or f_r) of the elements of G by $a \in G$ from the left (or right) is an isogroup $\widehat{G}$ with isomultiplication $\widehat{\times}$ defined on it. We say that $\widehat{G}$ is the* ***left-isotopically isomorphic realization*** *(or* ***right-isotopically isomorphic realization****) of the group G.*

Thus, G itself is the isotopically isomorphic realization (both left and right) of the group G where $a = e$.

Definition 6.2. *If G is a group then the set of all f_ls (or f_rs) corresponding to the left-isotopically isomorphic realizations (or right-isotopically isomorphic realizations) of G is called the* ***left-isopermutation group*** *(or* ***right-isopermutation group****).*

Definition 6.3. *A group for which both left-isopermutation group and right-isopermutation group coincide is called an* ***Iso-permutation group****.*

Remark:- 1) Note that if the group G is abelian then both left-isotopically isomorphic realizations and right-isotopically isomorphic realizations of G are same. Hence G is an isopermutation group.
2) Also, the left-isopermutation group of G is the left regular representation of G which is faithful. Similarly, the right-isopermutation group of G is the right-regular representation of G which is faithful.

Proposition 6.1. *Every left (or right) translation of a group G by an element $a \in G$ is an isogroup of first kind with induced iso-operation.*

Proof. It is clear that a^{-1} will act as the unity of the isogroup . The elements are lifted by a^{-1} and $x, y \in G$ imply $\hat{x}\hat{\times}\hat{y} = \hat{xay}$ as the induced iso-operation. □

Proposition 6.2. *The isomorphic realizations of a group G consist of a class of automorphisms of the group G and the class of translations of the group G by the elements from the group G itself.*

Proof. Aut(G) forms a class of permutations of G where identity is mapped with identity. Also, left regular representations and right regular representations form a class of permutations which are isotopically isomorphic to G. □

Let us consider the composition table of the abelian group $G = \{1, a, b, c\}$ with respect to multiplication.

·	1	a	b	c
1	1	a	b	c
a	a	1	c	b
b	b	c	a	1
c	c	b	1	a

Clearly, the set of left-isotopically isomorphic realizations of G = the set of right-isotopically isomorphic realizations of G = left regular representation of G = right regular representation of G

$$= \{ \begin{pmatrix} 1 & a & b & c \\ 1 & a & b & c \end{pmatrix}, \begin{pmatrix} 1 & a & b & c \\ a & 1 & c & b \end{pmatrix}, \begin{pmatrix} 1 & a & b & c \\ b & c & a & 1 \end{pmatrix}, \begin{pmatrix} 1 & a & b & c \\ c & b & 1 & a \end{pmatrix} \} < S_4.$$

Let us consider a non-abelian group,
$S_3 = \{e, f, g, h, i, j\} = \{(1), (23), (12), (123), (132), (13)\}$ with the following composition table;

·	e	f	g	h	i	j
e	e	f	g	h	i	j
f	f	e	h	g	j	i
g	g	i	e	j	f	h
h	h	j	f	i	e	g
i	i	g	j	e	h	f
j	j	h	i	f	g	e

Then,
set of left-Isotopically isomorphic realizations of G = left regular representation of G

$$= \{ \begin{pmatrix} e & f & g & h & i & j \\ e & f & g & h & i & j \end{pmatrix}, \begin{pmatrix} e & f & g & h & i & j \\ f & e & h & g & j & i \end{pmatrix}, \begin{pmatrix} e & f & g & h & i & j \\ g & i & e & j & f & h \end{pmatrix},$$
$$\begin{pmatrix} e & f & g & h & i & j \\ h & j & f & i & e & g \end{pmatrix}, \begin{pmatrix} e & f & g & h & i & j \\ i & g & j & e & h & f \end{pmatrix}, \begin{pmatrix} e & f & g & h & i & j \\ j & h & i & f & g & e \end{pmatrix} \} < S_6.$$

Proposition 6.3. *Permutations which represent the isotopically isomorphic groups of a group G form a subgroup of the group S_G.*

Proof. left-isotopically isomorphic realizations (or right-isotopically isomorphic realizations) of a group is the left regular representation (or right regular representation) of G and hence form a subgroup of S_G. □

Proposition 6.4. *If G is an abelian group of order n then the number of isotopically isomorphic realizations of G is n.*

Proof. Each element of the group will give rise to a left translation of the elements of G which is same as the right translation. □

Proposition 6.5. *If G is a non-abelian group of order n then the number of isotopically isomorphic realizations of G is $2n - 1$.*

Proof. Each element of G will give rise to distinct left and right translations of the group G. □

Proposition 6.6. *Holomorph of a group is the semidirect product of Aut(G) and left-isopermutation group (or right-isopermutation group) of G.*

Proof. Let G be a group and λ and ρ be the left regular and right regular permutation representation of G defined by $g^{\lambda} : x \to g^{-1}x$ and $g^{\rho} : x \to xg$. Clearly, both λ and ρ are regular and faithful representations of G. Thus, the left regular and right regular permutation representation of G form subgroups G^{λ} and G^{ρ} of Sym G. But G^{λ} = left isopermutation group of G and G^{ρ} = right isopermutation group of G. We know that [17], Hol $G = \langle G^{\lambda}, Aut(G)\rangle = \langle G^{\rho}, Aut(G)\rangle < Sym(G)$. Hence Hol $G = \langle$left isopermutation group, Aut $(G)\rangle = \langle$right isopermutation group, Aut$(G)\rangle < Sym(G)$. Also, Hol$(G) = Aut(G) \ltimes G^{\lambda}$ and Hol$(G) = Aut(G) \ltimes G^{\rho}$. Hence the result. □

Proposition 6.7. *If G is a group then the left-isopermutation group of G (right-isopermutation group of G) is the centralizer of right-isopermutation group of G (left-isopermutation group of G) in the holomorph of the group G.*

Proof. Let G be a group. Consider the left regular representation G^{λ} of G and the right regular representation G^{ρ} of G defined by $g^{\lambda} : x \to g^{-1}x$ and $g^{\rho} : x \to xg$. Then both λ and ρ are faithful representations of G. Also, the equations $C_{HolG}(G^{\rho}) = G^{\lambda}$ and $C_{HolG}(G^{\lambda}) = G^{\rho}$ hold for any group G. □

The work is in progress to cover additional relevant aspects of isopermutation group and will also be presented at ICNAAM 2014 in its Session 109 hosted by the R. M. Santilli Foundation being held at Rhodes, Greece during September 22-26, 2014.

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THE MASS OF THE NINTH RESONANCE IN THE PSEUDOSCALAR MULTIPLET

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Abstract

While the η' meson is a symmetrical combination of $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ chosen to be orthogonal to π^0 and η, its mass appears to be too large for the 0^- multiplet. To derive its mass, a formula for the 1^- multiplet combined with an octet mass for the 0^- multiplet is found to give a considerably more accurate estimate. A theoretical explanation of this η' state is provided. An equivalence is established between the formula for the mass and diagonalization of the mass matrix arising in a field theory containing a negative-parity gluon term.

Keywords: mass formula, gluonic state, vector dominance.

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1 Introduction

The η' meson mass is considered to be separate from the $SU(3)$ symmetry breaking mechanism for the pseudoscalar octet, and theoretical explanations for its value always have introduced new physical effects. The Goldsone boson that could arise from the axial symmetry of quantum chromodynamics [1], however, is known not to have sufficient mass and the branching ratios for the decays of the η and η' mesons to three pions prevent its identification with either of these particles [2]. The bag model of pseudoscalar mesons yielded an energy spectrum which approximated experimental values, although the additional constraint of parity would yield the energies tended to be higher than those of the lower-mass mesons [3]. The expansion of the correlator of four field strengths in terms of sums over meson states and glueball states revealed the existence of a meson of a mass determined by the non-zero value in the zero-momentum limit. The requirement of a vanishing second derivative of the energy with respect to the θ parameter, which is present in the spin-one Lagrangian with an instanton term and massless quarks, requires a mass for the η' meson that is proportional to $\frac{1}{\sqrt{N}}$ in the $\frac{1}{N}$ expansion. The limit $N \rightarrow \infty$ causes the anomaly in the axial current to vanish, and an identification with massless Goldstone boson can be made [4]. A related formula for the mass would yield a value of nearly 850 MeV with conventional values for the Goldberger-Treiman constant and the topological susceptibility in place of twice the second derivative of the energy with respect to the theta parameter. This value is generally combined with an octet mass in quadrature to give the η' mass. The η' meson also has been related to Ward identities resulting from axial symmetry yielding equations derived from a mass matrix including that of the η' meson [5].

The wavefunction of the η' meson

$$\eta' = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \tag{1}$$

is determined by its orthogonality to other states with $I_3 = S = 0$ in the 0^- multiplet. From the formula for the meson multiplets

$$M_{q\bar{q}} = m_1 + m_2 + A\frac{S_1 \cdot S_2}{m_1 m_2} \tag{2}$$

or

$$M_{q\bar{q}} = m_1 + m_2 - \frac{3A}{4m_1 m_2} \qquad J^P = 0^- \tag{3}$$

with $A = 0.0615\ (GeV/c^2)^3$, a mass is found for the η' meson that is significantly less than the experimentally measured value [6]. The spin-0 component in models of the η' had received some experimental support in the

examination of the reactions $\eta' \to \eta\pi\pi$ [7] and $\eta' \to \pi^+\pi^-\gamma$ [8]. There are other reactions, however, that reveal a connection with a vector component of the model [9].

It is verified in §2 that the 1^- multiplet formula is useful for deriving this particle's mass. An estimate of the widths for the reaction $\eta' \to \rho\gamma$ in §3 are found to consistent with the experimental values only when the coefficient for a vector rather than a pseudoscalar meson are used. Implications for the alignment of spins of the quarks in the η' meson shall be discussed in §4 in the context of both scattering experiments and decays and field theory.

The basis for a generalized model of the η' meson is conservation of parity and C-parity in the several of the reactions. This will require the increase of the angular momentum of the $q\bar{q}$ pair to one in these decays. The combination in quadrature with the octet mass then would yield the η' mass if the conservation of angular momentum alters this mass only infinitesimally. The octet mass then must correspond to a state with angular momentum equal to one, and this would be achieved through hyperfine splitting of the energy levels rather than the vector multiplet. This mechanism can occur because the quarks have fractional electric charge, and the higher energy is induced electromagnetically. Charge conjugation properties can be preserved through a superselection rule similar to that of the decay of positronium, where ${}^1S_0 \to \gamma\gamma$ and ${}^3S_1 \to \gamma\gamma\gamma$ are the allowed reactions. For the η' meson, the analogous states would consist of two and three gluons. A pseudoscalar two-gluon state is combined with other masses in the octet to give the η' mass in §5. Similarly the three-gluon state, which is genuinely nonperturbative, can generate a mass close to the experimental value.

2 Theoretical Prediction of the Mass of the η' Meson

The mass is defined in quantum field theory through the poles in the propagator that can receive perturbatuve corrections through quark-antiquark annihilation diagrams [10]. The momentum space Green function is

$$(4)\quad G_{\eta'} \sim \frac{1}{p^2+m_8^2} - \frac{m_0^2}{(p^2+m_8^2)^2} + \frac{1}{p^2+m_8^2} m_0^2 \frac{1}{p^2+m_8^2} m_0^2 \frac{1}{p^2+m_8^2} + \ldots$$

where

$$(5)\quad m_8^2 = \textit{average of the square of the } 0^- \textit{ octet masses}$$
$$m_0^2 = \textit{strength of the flavour singlet interaction.}$$

Since the pole is shifted from m_8^2 to $m_0^2 + m_8^2$ after the summation of the series,

$$(6)\quad G_{\eta'} \sim \frac{1}{p^2+m_8^2+m_0^2}.$$

Given broken $SU(3)$ symmetry,

(7) $$m_{\eta'}^2 = m_0^2 + m_8^2.$$

Since the derived mass from a 0^- multiplet does not match the experimental value, the possibility of the mass $m_{0,unmixed}$ being computed from the formula for the 1^- multiplet shall be considered. First, the spins of the quarks S_1 and S_2 are aligned such that $S_1 + S_2 = 1$, and

(8)
$$\begin{aligned} S_1 \cdot S_2 &= \frac{1}{2}((S_1+S_2)^2 - S_1^2 - S_2^2) \\ &= \frac{1}{2}(S(S+1) - S_1(S_1+1) - S_2(S_2+1)) \\ &= \frac{1}{2}(1\cdot 2 - \frac{1}{2}\cdot\frac{3}{2} - \frac{1}{2}\cdot\frac{3}{2}) \\ &= \frac{1}{4}. \end{aligned}$$

The value of A is verified by the prediction of the values of the other mesons in the pseudoscalar and vector multiplets. It follows that, for $\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$,

(9)
$$m_{0,unmixed} = \frac{1}{3}\left(2m_u + \frac{A}{4m_u^2}\right) + \frac{1}{3}\left(2m_d + \frac{A}{4m_d^2}\right) + \frac{1}{3}\left(2m_s + \frac{A}{4m_s^2}\right) = 863.96\ MeV/c^2$$

where the values of the constituent quark masses, $m_u = 310\ MeV/c^2$, $m_d = 310\ MeV/c^2$ and $m_s = 483\ MeV/c^2$ are used, and changes to m_d and m_s resulting from the mixing between generations have not been included. Furthermore, the octet of states in the 0^- multiplets yields

(10)
$$\begin{aligned} m_8 &= \frac{m_{K^+} + m_{K^-} + m_{K^0} + m_{\bar{K}^0} + m_{\pi^+} + m_{\pi^-} + m_{\pi^0} - + m_\eta}{8} \\ &= \frac{1}{8}(2(493.677\ MeV/c^2) + 2(497.614\ MeV/c^2) + 2(139.57\ MeV/c^2) \\ &\qquad + 134.9766\ MeV/c^2 + 547.853\ MeV/c^2) \\ &= 368.06895\ MeV/c^2 \end{aligned}$$

and

(11) $$m_{\eta'} = 939.096\ MeV/c^2.$$

The value (2.6) has been derived for the singlet state $\frac{1}{\sqrt{3}}(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$. Since the vector multiplet formula has been used for the mass, it cannot be identified with $|\eta_1\rangle$, although a connection with a gluon bound state shall become evident.

It is customary to include a mixing between the singlet and octet state $|\eta_8\rangle = \frac{1}{\sqrt{6}}(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle)$ upon SU(3) symmetry breaking. With the quadratic Okubo formula for m_{η_8} and a rotation of the squared matrix, the mass mixing is

$$|\eta'\rangle = cos\ \theta\ |\eta_1\rangle + sin\ \theta\ |\eta_8\rangle \tag{12}$$
$$|\eta\rangle = -sin\ \theta\ |\eta_1\rangle + cos\ \theta|\eta_8\rangle$$

where

$$|sin\ \theta^{(2)}| = \sqrt{\frac{m_{\eta_8}^2 - m_\eta^2}{m_{\eta'}^2 - m_\eta^2}} = \sqrt{\frac{(0.567)^2 - (0.547)^2}{(0.958)^2 - (0.547)^2}} = 0.1897877 \tag{13}$$
$$\theta^{(2)} \simeq 10.94°$$

while the linear equation defined by a rotation of the mass matrix gives

$$|sin\ \theta^{(1)}| = \sqrt{\frac{(0.567) - (0.547)}{(0.958) - (0.547)}} = 0.2205941 \tag{14}$$
$$|\theta^{(1)}| = 12.7439297°$$

which is close to the value of the Cabibbo angle.

This derivation can be refined if the mixing of quarks between different generations is included. It may be recalled that the amplitudes for weak interactions involving the coupling of the W boson and quarks receives weighted contributions from each generation, with the mixing described by the CKM matrix $\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$

[11]. Furthermore, in addition to the computation of the amplitudes, unitary of the CKM matrix implies that it is a norm-preserving transformation in a flavour space spanned by axes for each of the quarks. If it is postulated that the spinor space of the standard model is given by a direct sum of orthogonal components [12], the use of the axes representing eigenvectors of this matrix provides a new set of orthogonal states which are linear combinations of the d, s and b quarks. Nevertheless, these are physical states that can be included in the computation of cross sections for reactions.

Because the wavefunctions for the new quark state would be

$$d' = V_{ud}d + V_{us}d + V_{ub}b \tag{15}$$
$$s' = V_{cd}d + V_{cs}s + V_{cb}b$$
$$b' = V_{td}d + V_{ts}s + V_{tb}b,$$

for the meson multiplet,

(16)
$$\begin{aligned} m_u &= 310\ MeV/c^2 \\ m_{d'} &= |V_{ud}|^2 m_d + |V_{us}|^2 m_s + |V_{ub}|^2 m_s \\ &= 318.9561041\ MeV/c^2 \\ m_{s'} &= |V_{cd}|^2 m_d + |V_{cs}|^2 m_{cs} + |V_{cb}|^2 m_b \\ &= 473.7477405\ MeV/c^2 \end{aligned}$$

where the following absolute values of the CKM matrix elements [13][14][15]

(17)
$$\begin{aligned} |V_{ud}| &= 0.9738 \pm 0.0005 \\ |V_{us}| &= 0.2272 \pm 0.0030 \\ |V_{ub}| &= 0.0035 \pm 0.0015 \\ |V_{cd}| &= 0.224 \pm 0.012 \\ |V_{cs}| &= 0.97 \pm 0.16 \\ |V_{cb}| &= 0.0413 \pm 0.0015 \end{aligned}$$

are used. With the quark mixing,

(18)
$$m_0' = \frac{1}{3}\left(2m_u + \frac{A}{4m_u^2}\right) + \frac{1}{3}\left(2m_{d'} + \frac{A}{4m_{d'}^2}\right) + \frac{1}{4}\left(2m_{s'} + \frac{A}{4m_{s'}^2}\right)$$

(19)
$$\begin{aligned} &= \frac{1}{3}\left(2 \cdot 310\ MeV/c^2 + \frac{0.0615\ (GeV/c^2)^3}{4(0.310\ GeV/c^2)^2}\right) \\ &\quad + \frac{1}{3}\left(318.9561041\ MeV/c^2 + \frac{0.0615\ (GeV/c^2)^3}{4(0.3189561041\ GeV/c^2)^2}\right) \\ &\quad + \frac{1}{3}\left(2 \cdot (473.7477405\ MeV/c^2) + \frac{0.0615\ (GeV/c^2)^3}{4(0.4737477405\ GeV/c^2)^2}\right) \\ &= 861.6731178\ MeV/c^2. \end{aligned}$$

Furthermore,

(20)
$$\begin{aligned} m'_{\pi^0} &= 162.261071\ MeV/c^2 \\ m'_{K^+} &= m'_{K^-} = 469.6769598\ MeV/c^2 \\ m'_{\eta} &= 584.7395809\ MeV/c^2 \\ m'_{\pi^+} &= m'_{\pi^-} = m_u + m_{\bar{d}'} - \frac{3}{4}\frac{A}{m_u m_{\bar{d}'}} \\ &= 162.464569\ MeV/c^2 \\ m'_{K^0} &= m'_{\bar{K}^0} = m_{\bar{d}'} + m_{s'} - \frac{3}{4}\frac{A}{m_{\bar{d}'} m_{s'}} \\ &= 487.4500749\ MeV/c^2. \end{aligned}$$

Then mixing between generation yields an octet mass of

(21)
$$\begin{aligned} m'_8 = \frac{1}{8}(&2(469.67958\ MeV/c^2) + 2(487.450749\ MeV/c^2) + 2(162.464569\ MeV/c^2) \\ &+ (162.261071\ MeV/c^2) + 584.7395809\ MeV/c^2) \\ = 373.2738\ &MeV/c^2. \end{aligned}$$

Based on these values,

(22)
$$\begin{aligned} m'^2_{\eta'} &= m'^2_0 + m'^2_8 \\ &= (861.6731178\ MeV/c^2)^2 + (373.2738\ MeV/c^2)^2 \end{aligned}$$

and

(23)
$$m'_{\eta'} = 939.04947\ MeV/c^2$$

There is a nonperturbative contribution to the gluon and quark propagators [16].

(24)
$$\Pi^{\mu\nu}_{\eta'}(p^2) = iC^2_{\eta'} \int d^4x e^{ip\cdot x} \langle T\bar{q}(x)\gamma^\mu q(x)\bar{q}(0)\gamma^\nu q(0)\rangle_{vac} = (-g^{\mu\nu}p^2 + p^\mu p^\nu)\Pi_{\eta'}(p^2)$$

$$\Pi^{(N)}_{\eta'}(p^2) = \frac{\partial^N}{\partial(-p^2)^N}\Pi_{\eta'}(p^2)$$

$$\begin{aligned} \Pi^{(N)}_{NP} \approx \frac{3C^2_{\eta'}}{12\pi^2}(N-1)!\frac{1}{(-p^2)^N}\Bigg[1 + \frac{\hat{m}^2_q}{(-p^2)} - 4\pi^2\frac{N(N+1)}{p^4}m_q\langle q\bar{q}\rangle_{vac} \\ - \frac{3\pi N(N+1)}{8p^4}\langle\alpha_s G^2\rangle_{vac}\Bigg] \qquad N \geq 1 \end{aligned}$$

$$\Pi_{NP} \approx \frac{3C^2_{\eta'}}{12\pi^2}\left[1 + \frac{\hat{m}^2_q}{(-p^2)}\right] \qquad N = 0$$

With the values of $C_{\eta'}$, $\langle\bar{q}q\rangle$ and $\langle\alpha_s G^2\rangle$ [17][18][19], the nonperturbative terms can be added to the perturbative mass.

To estimate the correction, a matching of $\Pi_{NP}(p^2)$ and $\Pi(p^2)$ is required. Since

$$(25)\quad \Pi^{\mu\nu}_{NP} = -2\frac{C_\eta^2\left(\frac{1}{3}m_u\langle\bar{u}u\rangle + \frac{1}{3}m_d\langle\bar{d}d\rangle + \frac{1}{3}m_s\langle\bar{s}s\rangle\right)}{q^4}(-g^{\mu\nu}p^2 + p^\mu p^\nu)$$

has a singularity at $q = 0$ and

$$(26)\qquad \Pi_{NP}(p^2) = \frac{a}{m_{\eta'}^2 - p^2}$$

is divergent at $p^2 = m_{\eta'}^2$. To compare the two expressions, an intermediate value of p^2 is necessary.

Using the light-quark masses $m_u \simeq 4\ MeV$, $m_d = 7\ MeV$ and $m_s = 150\ MeV$, and the expectation values $\langle 0|\bar{u}u|0\rangle = (218\ MeV)^3$, $\langle 0|\bar{d}d|0\rangle = (238\ MeV)^3$ and $\langle 0|\bar{s}s|0\rangle = 1.44\left(\frac{\langle 0|\bar{u}u|0\rangle + \langle 0|\bar{d}d|0\rangle}{2}\right) = (257.8829\ mEv)^3$ [20], the nonperturbative correction to $m_{\eta'}$ may be evaluated.

Then, in units with $c = 1$,

$$(27)\qquad \Pi^{NP}(p^2 = \lambda m_{\eta'}^2) = \frac{1}{1-\lambda}\frac{a}{m_{\eta'}^2}$$

$$\Pi_{NP}(p^2 = \lambda m_{\eta'}^2) = \frac{-C_{\eta'}^2\left(\frac{1}{3}m_u\langle\bar{u}u\rangle + \frac{1}{3}m_d\langle\bar{d}d\rangle + \frac{1}{3}\langle\bar{s}s\rangle\right)}{\lambda^2 m_{\eta'}^2}$$

Since $(m_{\eta'}^P)^2 = \frac{1}{1-\lambda}\frac{a}{\Pi(p^2=\lambda m_{\eta'}^2)}$,

(28)

$$m_{\eta'} = \frac{1}{1-\lambda}\frac{a}{\Pi(q^2 = \lambda m_{\eta'}^2)}\left[1 + \frac{C_{\eta'}^2}{\Pi(p^2 = m_{\eta'}^2)}\frac{\left(\frac{1}{3}m_u\langle\bar{u}u\rangle + \frac{1}{3}m_d\langle\bar{d}d\rangle + \frac{1}{3}m_s\langle\bar{s}s\rangle\right)}{\lambda^2 m_{\eta'}^4}\right]$$

$$= (m_{\eta'}^P)^2\left[1 + \frac{1}{3}\frac{1-\lambda}{\lambda^2}\frac{\left(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle + m_s\langle\bar{s}s\rangle\right)}{m_{\eta'}^4}\right].$$

The coefficient λ required for the masses $m_{\eta'}^P$ and $m_{\eta'}$ satisfies

$$(29)\qquad \left[1 + 0.038423\frac{a}{(160\ MeV)^2}\frac{1}{C_{\eta'}^2}\frac{1-\lambda}{\lambda^2}\right]^{\frac{1}{2}} = 1.0202342$$

$$1.0638739\frac{a}{(160\ MeV)^2}\frac{1}{C_{\eta'}^2}\lambda^2 + \lambda - 1 \approx 0$$

with the positive root

$$\lambda = \frac{-1 + \sqrt{1 + 4(1.0638739)\frac{a}{(160\ MeV)^2}\frac{1}{C_{\eta'}^2}}}{2(1.0638739)\frac{a}{(160\ MeV)^2}\frac{1}{C_{\eta'}^2}}. \tag{30}$$

The intermediate scale, at which $\Pi(p^2)$ and $\Pi_{NP}^{\mu\nu}(p^2)$ can be matched, would be

$$\sqrt{\lambda} m_{\eta'}^P = \left[\frac{-1 + \sqrt{1 + 4(1.0638739)\frac{a}{(160\ MeV)^2}\frac{1}{C_{\eta'}^2}}}{2(1.0638739)\frac{a}{(160\ MeV)^2}\frac{1}{C_{\eta'}^2}}\right]^{\frac{1}{2}} m_{\eta'}. \tag{31}$$

3 Decays of the η' Meson

Consider the most prevalent decays of the η' meson [6]. These include

$$\eta' \rightarrow \eta\pi^+\pi^- \rightarrow \gamma\gamma\pi^+\pi^-\gamma\gamma\mu^+\nu_\mu\mu^-\bar{\nu}_\mu \rightarrow \gamma\gamma e^+\nu_e\bar{\nu}_\mu\nu_\mu e^-\bar{\nu}_e\nu_\mu\bar{\nu}_\mu$$
$$I(\eta') = 0\ I(\eta) = 0\ I(\pi^+) = 1\ I(\pi^-) = 1$$
$$P(\eta') = -1\ P(\eta) = -1\ P(\pi^+) = -1\ P(\pi^-) = -1$$
$$\eta' \rightarrow \rho^0\gamma \rightarrow \pi^+\pi^-\gamma \rightarrow \mu^+\nu_\mu\nu^-\bar{\nu}_\mu\gamma \rightarrow e^+\nu_e\bar{\nu}_\mu\nu_\mu e^-\bar{\nu}_e\nu_\mu\bar{\nu}_\mu\gamma$$
$$I(\eta') = 0\ I(\rho^0) = 1\ I(\gamma) = 1$$
$$P(\eta') = -1\ P(\rho^0) = -1\ P(\gamma) = -1$$
$$\eta' \rightarrow \eta\pi^0\pi^0 \rightarrow \gamma\gamma\gamma\gamma\gamma\gamma\gamma$$
$$I(\eta') = 0\ I(\eta) = 0\ I(\pi^0) = 1$$
$$P(\eta') = -1\ P(\eta) = -1\ P(\pi^0) = -1$$
$$\eta' \rightarrow \omega\gamma \rightarrow \pi^+\pi^0\pi^-\gamma \rightarrow \mu^+\nu_\mu\gamma\gamma\mu^-\bar{\nu}_\mu\gamma \rightarrow e^+\nu_e\bar{\nu}_\mu\nu_\mu\gamma\gamma e^-\bar{\nu}_e\nu_\mu\bar{\nu}_\mu\gamma$$
$$I(\eta') = 0\ I(\omega) = 1\ I(\gamma) = 0$$
$$P(\eta') = -1\ P(\omega) = -1\ P(\gamma) = -1$$
$$\eta' \rightarrow \gamma\gamma$$
$$I(\eta') = 0\ I(\gamma) = 0$$
$$P(\eta') = -1\ P(\gamma) = -1$$
$$\eta' \rightarrow 3\pi^0 \rightarrow \gamma\gamma\gamma\gamma\gamma\gamma$$
$$I(\eta') = 0\ I(\pi^0) = 1$$
$$P(\eta') = -1\ P(\pi^0) = -1$$

For the reactions $\eta' \rightarrow \rho^0\gamma$, $\eta' \rightarrow \omega\gamma$ and $\eta' \rightarrow \gamma\gamma$, parity is not conserved apparently unless there is a relative parity for the final products or an additional contribution to the angular momentum. Furthemore, there is a change in the total isospin, $|\Delta I| = 1$ in the first two decays. The solution to this parity problem is given after an analysis of the width of one of the decays of the η' meson. For $\eta' \rightarrow \rho^0 + \gamma$, considered initially as a decay from a pseudoscalar meson to a vector meson and a photon, an expression for partial width of the resonance has been derived [21].

$$\Gamma(P \rightarrow V + \gamma) = 4\alpha|\langle V|\hat{\mu}_z|P\rangle|^2 k^3\Omega^2 \tag{32}$$

where α is the fine structure constant, $\hat{\mu}_z = \sum_i \left(\frac{Q_i}{2m_ic^2}\right)\sigma_{iz}$, k is the photon momentum, and Ω is an overlap integral representing a phase factor. Since

$$|\eta'\rangle = sin\ \phi|\eta_{ns}\rangle + cos\ \phi|\eta_s\rangle \tag{33}$$

with $|\eta_{ns}\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)$ and $|\eta_s\rangle = |s\bar{s}\rangle$, $\phi = 43.7356°$ and $|\rho^0\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle)$, such that

(34)
$$\langle\rho^0|\hat{\mu}_z|\eta_{ns}\rangle = -\frac{1}{6}\left(\frac{2}{m_u} + \frac{1}{m_d}\right)$$
$$\langle\rho^0|s\bar{s}\rangle = 0$$

and

(35)
$$\Gamma_{P\to V+\gamma}(\eta' \to \rho^0 + \gamma)|_{\theta=11°}(max.) = 4\frac{1}{137}\cdot(0.69133)^2\left(\frac{1}{3\cdot 336\ MeV} + \frac{1}{6\cdot 336\ MeV}\right)^2 (kc)^3\Omega^2(max.)$$
$$= 165.033\ keV$$

because $\Omega^2(max.) = \frac{m_\rho}{m_{\eta'}}$. If the mixing angle $\tilde{\varphi} = arc\ tan(\sqrt{2}) + 11°$ is used, then

(36)
$$\Gamma_{P\to V+\gamma}(\eta' \to \rho + \gamma)|_{\theta=11°}(max.) = 4\cdot\frac{1}{137}\cdot(0.91166)^2\left(\frac{1}{3\cdot 336} + \frac{1}{6\cdot 336}\right)^2 (188)^2\Omega^2(max.)MeV$$
$$= 286.98834\ keV$$

The difference can be removed if the factor of 4α, resulting from a sum over spins for the pseudoscalar, is replaced by $\frac{4\alpha}{3}$ for an average over spins for a vector meson. Division of the last value by a factor of 3 gives $95.65947\ keV$.

These values may be adjusted according to the factor

(37)
$$(m_{\eta'}c^2 - E_\rho)^3\left(\frac{E_\rho}{m_{\eta'}c^2}\right)(m_{\eta'}c^2 - m_\rho c^2)^3\left(\frac{m_\rho}{m_{\eta'}}\right) = \frac{\left(1 - \frac{E_\rho}{m_{\eta'}c^2}\right)^3 E_\rho}{\left(1 - \frac{m_\rho}{m_{\eta'}}\right)^3 m_\rho c^2}$$

When $\phi = 43.3576°$, coincidence with the experimental value is attained at $E_\rho = 818\ MeV$, and, if $\tilde{\varphi} = 65.3576°$, equality is achieved at $E_\rho = 843\ MeV$. With a factor in the formula for the width corresponding to a spin-one particle, there is agreement between the theoretical and experimental widths at $E_\rho = 791\ MeV$.

This result provides support for the existence of an additional component in the η' meson related to the vector multiplet. Parity is conserved because a new factor of -1 is introduced through $(-1)^{L+1}$ for the 1^- state. In connection with the conservation of parity and C-parity in this reaction, it can be demonstrated now since $P = (-1)^2 = 1$, and the parities of the

initial and final states are equal. The C-parity $(-1)^{L+S}$ of the η' meson is 1, while that of ρ^0 and γ is equal to -1.

4 Alignment of Spins of the Quarks in the η' Meson

While the octet contribution was defined by an average of the squared masses of the particles in the 0^- multiplet, the other flavour-singlet term has been linked to the 1^- multiplet. Since the primary identification of the η' meson is the flavour singlet, characterization of its spin would be an inherent property of the particle.

An examination of the reactions that produce the η' meson reveals that it could be described by a state with either spin 0 or 1. Examples include the decay of exotic meson states with $J^{PC} = 1^{-+}$, created in $\pi^- p$ and pN scattering, to $\eta'\pi^+$ [22][23] and the reaction $\gamma p \rightarrow p\eta'$ [24]. One resolution of this problem is the view of the spin-1 particle as an excited state of the spin-0 particle, with the alignment of spins of the quarks and anti-quarks in the ground state having opposite directions. In general, the alignment of spins in opposite directions would be energetically favoured, as the masses of vector mesons are generally larger than those of the pseudoscalar mesons. However, experimental evidence of a particle satisfying the formula for the 0^- multiplet (1.3) has not been reported.

Furthermore, the $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ states, as virtual pairs, could annihilate and create a photon or gluon with spin 1 [24]. If the annihilation of the quarks and anti-quarks occurred before an alignment of spins in opposite directions, the state would appear as a particle with $J^P = 1^-$ which essentially had the form of a photon or a gluonic bound state. This concept is supported by the observation that the η' is identified actually through the missing mass technique. Furthermore, near-threshold production of η' in the nucleon interactions is dependent on a large momentum transfer at approximately 0.3 fm [25], which is indicative of quark-gluon degrees of freedom in the dynamics.

A comparison of $pp \rightarrow pp\eta'$ and $pn\eta' \rightarrow pn\eta'$ cross sections reveals vector dominance [26][27]. Since the protons must exist in different spin states, the total isospin is 0, whereas the pn system can have total isospin equal to 0 or 1. Then $\sigma(pp \rightarrow pp\eta') = \sigma_{I=0}$ and $\sigma(pn \rightarrow pn\eta') = \frac{\sigma_{I=0}+\sigma_{I=1}}{2}$. If $R_{\eta'} = \frac{\sigma(pn\rightarrow pn\eta')}{\sigma(pp\rightarrow pp\eta')}$, it follows that $\sigma_{I=1} = (2R_{\eta'} - 1)\sigma_{I=0}$, which favours the $I = 1$ channel for $R_{\eta'} > 1$.

While it is conventional to consider the two orthogonal states

(38)
$$\eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$
$$\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

it is the flavour singlet that can be transformed into a photon or a gluonic bound state. Furthermore, a gluon has the required quantum numbers, but it does not exist as a free state by confinement.

The necessity of a bound state of gluons therefore follows. However, the spin and the parity of this bound state then also must be 1^-, which requires alignment of one pair of spins in opposite directions. The coupling of the gluon to photons in this channel has been shown to be necessary for the decay $\eta' \rightarrow \gamma\gamma$ [28]. A composite state of three gluons, for example, must have two gluons with opposite spins to be included in the description of the η' meson.

The possibility of a flavour-singlet state of the η' meson consisting entirely of a glueball is partially corroborated by computations with the bag model of the baryons, mesons and gluon bound states. The lowest mass glueball state is known to be the transverse electric component $(TE)^2$ with quantum numbers 0^{++} [3][29]. It has mass 960 MeV/c^2 and the parity would be consistent with those of the reactions $\eta' \rightarrow \rho^0\gamma$, $\eta' \rightarrow \omega\gamma$ and $\eta' \rightarrow \gamma\gamma$ in §3. While this mechanism might not be required if there had been other particles in any of these reactions, it is possible also to consider the $J = 1$ states for the purposes of consistency with the previously described experimental results.

The three-gluon states also could be identified in the lattice gauge theory with three-link paths for Kogut-Susskind quarks [30][31][32]. The quadratic formula for the η' mass depends on the combination of a quark propagator with mass m_8 and a gluon subdiagram with mass m_0. Since the mass m_8 has been derived from the 0^- multiplet, angular momentum is conserved at each vertex for a diagram with a pseudoscalar glueball. For the three-gluon bound state, angular momentum is conserved if the quark-anti-quark pairs are vector states. The octet mass for the 1^- multiplet, omitting the φ meson,

(39)
$$\begin{aligned} m_8(1^- \textit{ multiplet}) &= \frac{1}{8}(2(891.66\ MeV/c^2) + 2(895.94\ MeV/c^2) + 2(775.11\ MeV/c^2) \\ &\quad + 775.49\ MeV/c^2 + 782.65\ MeV/c^2) \\ &= 835.445\ MeV/c^2. \end{aligned}$$

is too large, however, to be used in the quadratic formula, and an excited state with a mass closer to $m_8(0^-\ multiplet)$ is required.

It is known that pseudovector mesons with $J^P = 1^{+-}$ have considerably larger masses [6]. Hyperfine splitting of quarkonium derived from the strong interaction potential is given by $[M(^3S_1)]^2 - M[(^1S_0)]^2 = \kappa\frac{64\pi\alpha_s(\mu)|\psi(0)|^2}{9\mu}$, where μ is the reduced mass of the quark-anti-quark pair and κ is found from the value of $|\psi(0)|^2$ and the colour magnetic moment, although mass differences predicted by this mechanism tend range from 49.7 MeV/c^2 to 652 MeV/c^2 for various quarkonium states when $\Lambda = 160\ MeV$ in the $1S$ state [33]. The quark-anti-quark pair also has fractional electric charge and the formation of an electromagnetically bound state equivalent to positronium would produce a hyperfine splitting in the energy levels. The electromagnetic corrections to the masses of mesons have been calculated and found to equal several MeV [34]. Nonperturbative corrections can be added to the electromagnetically excited energy level to give the mass of the η' meson.

It has been suggested that the inclusion of quark-annihilation diagrams and two-gluon states in the bag model is consistent with the higher value of η'-meson mass [35]. While the bag model for mesons might could be considered for the η' meson, quark predictions of the baryon magnetic moments are more accurate than those of the bag model which describes glueball states [36]. With the quarks located interior to the boundary, the bag model would not necessarily predict the characteristics of quark configurations. By contrast, the nonperturbative bound states of gluons consistent with the slopes of the pomeron trajectories [37], which can be viewed as dual to perturbative gluon vertices, may be described by a bag model rather than a string model.

5 On the Derivation of the η' Mass through Field Theory

An alternative derivation of m_0 through instantons [4][38][39] [40]

$$m_0^2 = 4\frac{N_f\chi}{f_\pi^2} \tag{40}$$

where

$$\begin{aligned} N_f &= number\ of\ flavours \\ \chi &= topological\ susceptibility \\ f_\pi &= pion\ decay\ constant \end{aligned} \tag{41}$$

follows again from the summation of quark-anti-quark loops. In configuration space, this equals

$$\frac{1}{V}\sum_{x,y} H(x,y) = \frac{\langle Q^2\rangle}{V m_q^2} = \frac{\chi}{m_q^2} \tag{42}$$

with Q being the number of zero modes of the quark Dirac operator. The average over the quark flavours of the amplitude, in momentum space, equals

$$H(q) = f_P \frac{1}{q^2+m_\pi^2}\frac{m_0^2}{N_f}\frac{1}{q^2+m_\pi^2} f_P \tag{43}$$

$$f_P = \langle 0|\bar{\psi}\gamma_5\psi|0\rangle = \frac{m_\pi^2 f_\pi}{2m_q}.$$

The equality between Eq.(5.3), in the large-volume limit, and Eq. (5.4), when $q = 0$, gives Eq.(5.1). This relation had been derived in a $\frac{1}{N_c}$ expansion [4][41], and the limit $N_c \to \infty$ is replaced by these limits in configuration and momentum space.

The mass m_0 then has been identified as a quenched approximation to $m_{\eta'}$ in the chiral limit. From the discussion in §2 and §3, this interpretation also follows from the limit of the vanishing of the pseudoscalar meson masses, which would imply $m_8 \to 0$, leaving a contribution to $m_{\eta'}$ separate from the 0^- multiplet.

The original form of Eq.(5.1), based on a cancellation of a leading-order glueball contribution and that of mesons to

$$U(k) = \int d^4x e^{ik\cdot x}\langle T F(x)\tilde{F}(x)F(0)\tilde{F}(0)\rangle \tag{44}$$

which could be complemented by a summation of the perturbation series in the limit $k \to 0$, includes $2\left(\frac{d^2E}{d\theta^2}\right)_{\theta=0}^{no\ quarks}$ [4] in place of χ and $f_{\eta'}$ may be substituted for f_π. There is a coincidence between $\left[\left(\frac{d^2E}{d\theta^2}\right)_{\theta=0}^{no\ quarks}\right]^{\frac{1}{4}}$, when the formula contains f_π, with several well-established values in QCD including the fourth root of the computed pressure against the boundary of the bag model of the pseudoscalar meson [35], which could be replaced by that for the glueball bound state, and the contribution to the mass matrix for the 0^- multiplet resulting from quark-anti-quark annihilation into gluons. In the formula for the pseudoscalar meson masses, the mass matrix based on the one-gluon exchange potential are not valid for m_η and

$m_{\eta'}$ [20]. In the $\{\pi_0, \eta_{ns}, \eta_s\}$ basis

$$(45) \qquad \begin{pmatrix} m_\pi & 0 & 0 \\ 0 & m_\pi + 2A_{q\bar{q}} & \sqrt{2}A_{q\bar{q}} \\ 0 & \sqrt{2}A_{q\bar{q}} & 2m_K - m_\pi + A_{q\bar{q}} \end{pmatrix}$$

and diagonalization yields

$$(46) \qquad m_\eta m_{\eta'} = m_\pi(2m_K - m_\pi) = A_{q\bar{q}}(4m_K - m_\pi)$$
$$m_\eta + m_{\eta'} = m_{\eta_{ns}} + m_{\eta_s} = 2m_K + 3A_{q\bar{q}}$$

which requires $A_{q\bar{q}} = 172\ MeV$. Furthermore, for a ${}^3S_1\ q\bar{q}$, there would be a contribution from the three-gluon exchange.

With the inclusion of the $\theta G\tilde{G}$ term in the Lagrangian, which is necessary for the generating functional of the transition amplitudes between different θ-vacua [8], it can be seen that a modified $U(1)$ current is conserved non-perturbatively. It appears to be preferable to use the original axial current to solve the problem of the absence of the $U(1)$ Goldstone boson and the non-zero branching for $\eta \to 3\pi$ [1][2]. However, the breaking of this symmetry also could be achieved through the introduction of another term, in a manner similar to that of the bag model, provided renormalizability can be restored.

A further indication of contributions to $m_{\eta'}$ from two different sectors is provided by the model based on the addition of a gluon term with the opposite parity to the Lagrangian, ΘQ, where

$$(47) \qquad Q = \sqrt{2N_f}\frac{\alpha_s}{8\pi}\frac{1}{2}\epsilon_{\mu\nu\lambda\rho}G^{A\mu\nu}G^{A\lambda\rho}$$

and Θ is a coupling [42]. It is found that the $U(1)$ charge in the effective Lagrangian, which must be chirally and renormalization-group invariant, cannot scale with the correct power of the number of colours, N_c, unless another gluon field η^G is included in addition to initial singlet η^0 and octet interpolating fields [42]. Furthermore, gluon bound states cannot be linked to the breaking of the global $SU(3)$ symmetry and, consequently, m_8. The two contributions to $m_{\eta'}$ can be traced to the addition of the negative-parity glueball term to the QCD Lagrangian.

In the basis of states η^8, η^0, η^G, the squared mass matrix, deduced from the effective Lagrangian, is

$$(48) \qquad M^2 = \begin{pmatrix} \frac{1}{3}(4M_K^2 - M_\pi^2) & \frac{2\sqrt{2}}{3}r(M_\pi^2 - M_K^2) & 0 \\ \frac{2\sqrt{2}}{3}r(M_\pi^2 - M_K^2) & M_\eta^2 + \frac{\beta_0}{3}(2m_K^2 + M_\pi^2) & q \\ 0 & q & M_G^2 \end{pmatrix}.$$

with $r = \frac{F}{F^0}$, where F is the universal decay constant for the octet of mesons and $F^0 = \frac{\langle\bar{u}u\rangle_0}{\langle 0|J_5^0|\eta_5^0\rangle}$ arises in a $U(1)_A$ transformation of the η^0 field,

while $\beta_0 = r^2$ [42]. When $\beta_0 = r = 1$, the eigenvalue equation would be

(49)

$$\lambda^3 - (2m_K^2 + m_\eta^2 + m_G^2)\lambda^2 + \{[\frac{1}{3}(4m_K^2 - m_\pi^2)][m_\eta^2 + \frac{1}{3}(2m_K^2 + m_K^2)]$$
$$+ [\frac{1}{3}(4m_K^2 - m_\pi^2)]m_G^2 + [m_\eta^2 + \frac{1}{3}(2m_K^2 + m_\pi^2)]m_G^2 - q^2 - \frac{8}{9}(m_\pi^2 - m_K^2)^2\}\lambda$$
$$- [\frac{1}{3}(4m_K^2 - m_\pi^2)][m_\eta^2 + \frac{1}{3}(2m_K^2 + m_\pi^2)]m_G^2 + \frac{q^2}{3}(4m_K^2 - m_\pi^2) + \frac{8}{9}(m_\pi^2 - m_K^2)m_G^2$$
$$= 0.$$

With a shift $\lambda = y + \frac{1}{3}(2m_K^2 + m_\eta^2 + m_G^2)$,

(50) $$y^3 + p'y + q' = 0$$

where

(51)

$$p' = -\frac{4}{3}m_K^4 + \left(\frac{2}{3}m_G^2 + 2m_\pi^2\right)m_K^2 - q^2 + [m_\eta m_G^2 - \frac{1}{3}(m_\eta^2 + m_G^2)^2 - \frac{1}{3}m_\pi^2 m_\eta - m_\pi^4]$$

$$q' = \left[\frac{1}{9}(4m_K^2 - m_\pi^2)m_\eta^2 + \frac{2}{3}m_K^2 m_\pi^2 - \frac{1}{3}m_\pi^4 + \frac{1}{27}(10m_K^2 + 2m_\pi^2)m_G^2 + \frac{1}{3}m_\eta^2 m_G^2\right]$$
$$\cdot (2m_K^2 + m_\eta^2 + m_G^2) + \frac{q^2}{3}(2m_K^2 - m_\eta^2 - m_G^2 - m_\pi^2) - \frac{1}{3}(4m_K^2 - m_\pi^2)m_0^2 m_G^2$$
$$+ \left[\frac{19}{27}m_\pi^4 - \frac{38}{27}m_K^2 m_\pi^2 - \frac{8}{27}m_K^4\right]m_G^2 - \frac{8}{27}(m_K^2 - m_\pi^2)^2(2m_K^2 + m_\eta^2).$$

From the values of m_K, m_{η^0}, m_G and m_π respectively, it follows that the root

(52) $$r_1 = \left[-\frac{q'}{2} + \sqrt{\frac{q'^2}{4} + \frac{p'^3}{27}}\right]^{\frac{1}{3}} + \left[-\frac{q'}{2} - \sqrt{\frac{q'^2}{4} + \frac{p'^3}{27}}\right]^{\frac{1}{3}}$$

is approximately equal to the square of $m_{\eta'}$ when $q^2 = 0.6(GeV/c^2)^4$. Because, $\left(\frac{q'}{2}\right)^2 + \left(\frac{p'}{3}\right)^3 < 0$,

(53) $$r_1 \approx 2\ cos\left(\frac{55.432751^\circ}{3}\right)\left(\left|\frac{p'^3}{27}\right|\right)^{\frac{1}{6}} = (1.896895449)\left|\frac{p'}{3}\right|^{\frac{1}{2}}$$

which implies an algebraic relation

(54)

$$m_{\eta'}^2 \approx 1.095082836 \cdot \left[\frac{4}{3}m_K^4 - \left(\frac{2}{3}m_G^2 + 2m_\pi^2\right)m_K^2 + \left[\frac{1}{3}(m_\eta^2 + m_G^2)^2 + \frac{1}{3}m_\pi^2 m_\eta^2 + m_\pi^2 - m_\eta^2 m_G^2\right] + \frac{3}{5}(GeV/c^2)^4\right]^{\frac{1}{2}}$$

complementing the the Witten-Veneziano formula [4][41]

(55) $$m_{\eta'}^2 + m_\eta^2 - 2m_K^2 = m_0^2.$$

where m_0 is identified with m_G.

6 Conclusion

The flavor-singlet mass, m_0, is added in quadrature with the octet mass m_8 to give $m_{\eta'}$. The octet mass is chosen to be the average of the masses of the non-singlet pseudoscalar mesons and the quark propagator in the quark-anti-quark annihilation diagrams are combined with a pseudoscalar flavour-singlet state of the η' meson. With the addition of a topological term not conserving parity, the existence of a pseudoscalar two-gluon state would follow. Another formula for $m_{\eta'}$ resulting from the diagonalization of the mass matrix with a field η^G in the §5 has yielded a mass for the two-gluon state that coincides with m_0.

There are reactions, however, described in §2 and §3, which have widths that are considerably less than those predicted with an initial pseudoscalar state. The theoretical and experimental values of the widths [6] are compatible, however, if the process is the decay of an excited η' in a vector state. Consequently, the possibility of a spin-one component of the η' field, and the temporary alignment of the spins of the quarks and anti-quarks, must be considered. These states can decay into three gluons. However, when the flavor-singlet state of the η' consists of three gluons, conservation of angular momentum in the quark-anti-quark annihilation diagrams requires that the octet mass represents a vector state. Although the average of the masses of the 1^- multiplet is too large for m_8, the difference from an excited electromagnetically bound state of the linear combination of quarks and anti-quarks would be negligible. Given a hyperfine splitting of the energy levels, the new octet mass could be used in the quadratic formula with the value of m_0 deduced from the 1^- spin-coupling formula to produce $m_{\eta'}$.

In the general model of the η' meson, pseudoscalar two-gluon states occur in many of the reactions. For the remainder of the reactions, the vector three-gluon states would be sufficient for conservation of angular momentum and the relative widths. The experimental verification of the existence of

these flavour-singlet states of the η' meson and the hyperfine splitting of electromagnetically bound quark-anti-quark states would provide a basis for this theoretical explanation of the η' meson mass.

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DIMENSIONLESS PHYSICAL CONTANT MYSTERIES (III) THE ANOMALY OF LEPTONS AND BARYONS

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Abstract

From $|\alpha^{1/2}| = \log e^{\pm\phi/K\pi}$, $\alpha^{-1} = 137.03600038513325888$, we get the g-2 factors of particles (Leptons and Baryons)

$$\frac{1}{2}\begin{pmatrix} |g_e| & |g_p| \\ |g_\mu| & |g_n| \end{pmatrix} = \begin{pmatrix} \mathbf{1.0011596521807} & \mathbf{2.792847356} \\ \mathbf{1.0011659207972} & \mathbf{1.913042720} \end{pmatrix} =$$

$$\begin{pmatrix} \ln\left(\frac{\exp\left[\frac{\alpha^3}{2\pi}(\alpha^{-2}+\frac{2}{3})\right]}{\exp\left[(\frac{\alpha}{\pi})^2(\alpha^2+\frac{1}{3})\right]}e^{1+\mathcal{O}_e}\right) & \ln\left(\frac{\exp\left[\frac{1}{3}(\frac{11}{9\pi}\alpha^{1/2})\right]}{\exp\left[\frac{1}{\pi}(8\alpha^{1/2})\right]}e^{3+\mathcal{O}_p}\right) \\ \ln\left(\frac{\exp\left[\frac{\alpha}{2\pi}(\frac{2\alpha}{\pi}+1)\right]}{\exp\left[\frac{3\pi\alpha^3}{\pi+1}\right]}e^{1+\mathcal{O}_\mu}\right) & \ln\left(\frac{\exp\left[\frac{2}{3}(\frac{11}{9\pi}\alpha^{1/2})\right]}{\exp\left[\frac{1}{\pi}(4\alpha^{1/2})\right]}e^{2+\mathcal{O}_n}\right) \end{pmatrix}$$

Keywords: Fine structure constant, Quantum Theory, g-factors, Lepton, Baryons, Spin, S-matrix.

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1 Introduction

The spin magnetic moment of a particle i is $\mu_s^i = \frac{|g_i|}{2}\frac{\mathbf{e}}{m_i}s$, where $\mathbf{e}$, m_i, s and $|g_i|$ are the particle's charge, mass, spin and g-factor. [1] We have discussed charge and mass with the α formula in previous papers (I) and (II). [2,3] Next, we discuss the spin and the g-factor of particles. These two physical properties are mainly linked with the magnetic field.

There is no special particle for the magnetic force in nature, the macro-magnetic phenomena is due to electron motion, either linear or circular. The charge and mass of a particle are studied as the electrostatic and gravitational forces, both central forces, as Coulomb's law and Newton's law. There are parallels between the electrostatic and gravitational fields. The Coulomb law, which describes the interaction of two electric charges, $\mathbf{F} = q(k_eQ\hat{\mathbf{r}}/|\mathbf{r}|^2) = q\mathbf{E}$, is similar to the Newton law of universal gravitation describing the interaction of two masses, $\mathbf{F} = m(GM\hat{\mathbf{r}}/|\mathbf{r}|^2) = m\mathbf{g}$. Due to the similarities between the electric field $\mathbf{E}$ and the gravitational field $\mathbf{g}$, sometimes mass is called "gravitational charge".

The magnetic field is different from the electric field with north/south poles; due to electric charge motion being a vector with a starting and ending point, there is no monopole "magnetic charge" owing to the vector property. The electric charge motion is described by the Lorentz force law as $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$. The magnetic dipoles are attracted/repelled into regions of higher magnetic fields. The force on a small magnet having a magnetic moment μ due to a magnetic field $\mathbf{B}$ is $\mathbf{F} = \nabla(\mu \cdot \mathbf{B}) = \nabla(\mu B \cos\theta)$, where the gradient ∇ is the change of the quantity $\mu \cdot \mathbf{B}$ per unit distance and the direction is that of maximum increase of $\mu \cdot \mathbf{B}$.

The Schrödinger equation $[\frac{-\hbar^2}{2m}\nabla^2 - V(\mathbf{r})]\Psi(\mathbf{r},t) = i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)$ only concerns the electrostatic force in the central potential $V(\mathbf{r}) = -\frac{Z\mathbf{e}^2}{r}$. It can not yield the fine structure solution, having no spin-1/2 quantum number, and is therefore incomplete, as Einstein pointed out. The Dirac relativistic equation $[\sum_{k=1}^{3}\alpha_k \cdot P_k c + \beta \cdot mc^2]\Psi_{1,2,3,4} = i\hbar\frac{\partial}{\partial t}\Psi_{1,2,3,4}$ adds the $E = mc^2$ term with four 4×4 spinor matrices, automatically yielding the spin-1/2 and the Sommerfeld fine structure, but still can not calculate

the electron anomaly $a_e \simeq \frac{\alpha}{2\pi}$ proposed by Schwinger. Therefore, due to magnetic force being treated separately from electric force in quantum theory, the electron perturbation became a new problem. QED worked on the problem of electron perturbation, and is still incomplete on the electron anomaly. Again, Einstein is correct in the argument of quantum theory, it is not an accomplished mission.

The spin and g-factor of particles are also related to α. However, there is no general simple formula can calculate the g-factor of particles. We will discuss these issues by the formula of the fine structure constant [2]

$$|\alpha^{1/2}| \equiv \pm\frac{M\phi}{K\pi} \equiv \frac{\log e^{\pm 1}}{K\Phi\pi} \equiv \log e^{\pm\phi/K\pi} \equiv \frac{1}{\ln 10^{\pm K\Phi\pi}} \tag{1}$$

where $K(J,H) = 1 + \frac{1^H}{J+}\frac{3^H}{J+}\frac{5^H}{J+}\frac{7^H}{J+}\cdots$, $J = 6771$, $0 \leq H < \alpha^{-1/2}$, and $\Phi - \phi - e - \pi$ in Euler-type Identity $e^{\pm i\pi} + \Phi = \phi$. If $H = 0$, $K(J) = [1, \overline{6771}] = 1 + \frac{-6771+\sqrt{6771^2+4}}{2} = 1.000147688669057459316502179743 8$, which yield $\alpha^{-1} = 137.036000385133258885821578824 37$. This theoretical value is used in following calculation.

2 Spin and Fine Structure Constant

Pauli suggested the connection between spin and statistics by the *odd-even* numbers. [4] He also wondered at *odd-even* or *prime-even* numbers when searching for the fine structure constant formula. [4]

Bose-Einstein distribution linked to the *even* numbers can be derived from [5]

$$\frac{\sum_{n=0}^{\infty}(+1)^n(n+1)x^{n+1}}{\sum_{n=0}^{\infty}(+1)^n x^n} = \frac{\sum_{n=0}^{\infty}(+1)^n n e^{nh\nu/kT}}{\sum_{n=0}^{\infty}(+1)^n e^{h\nu/kT}} = \frac{1}{e^{h\nu/kT}-1} \tag{2}$$

where $x = \exp(-E/kT)$, i.e., $E = h\nu = -kT\ln x$ for the boson with spin $s = \frac{n}{2}\hbar$ $(n = 2, 4, 6, ..., even)$, such as photon or phonon.

A Fermi-Dirac distribution linked to the *odd* numbers can be derived from [6, 7]

$$\frac{\sum_{n=0}^{\infty}(-1)^n(n+1)x^{n+1}}{\sum_{n=0}^{\infty}(-1)^n x^n} = \frac{\sum_{n=0}^{\infty}(-1)^n n e^{\pm nh\nu/kT}}{\sum_{n=0}^{\infty}(-1)^n e^{\pm h\nu/kT}}$$
$$= \frac{1}{e^{h\nu/kT}+1} \tag{3}$$

where $x = \exp(\pm E/kT)$, i.e., $E = mc^2 = \pm kT \ln x$ for the fermion with spin $s = \frac{n}{2}\hbar$ ($n = 1, 3, 5, ..., odd$), such as electron/positron, proton and neutron. Unlike the boson in Bose-Einstein distribution which always has a negative chemical-potential (like the gravitational field), a fermion in Fermi-Dirac distribution can have both + or − chemical-potential (like the electrostatic fields).

Combining (2) and (3), we have

$$\frac{\sum_{n=0}^{\infty}(\pm 1)^n(n+1)x^{n+1}}{\sum_{n=0}^{\infty}(\pm 1)^n x^n} = \frac{x}{1 \mp x} = \frac{1}{e^{E/kT} \mp 1} \tag{4}$$

where $(\pm 1)^n$ gives the "+" for Bose-Einstein distribution and "−" for Fermi-Dirac distribution; and *odd-n* terms have the alternative ± sign and *even-n* terms only have + sign. Since $x = \exp(\pm E/kT)$, we have $\log x = \log[\exp(\pm E/kT)]$. This is same format as $\alpha^{1/2} = \log[\exp(\pm\phi/K\pi)]$ $= \frac{M\phi}{K\pi}$, and we should remember the Wallis formula (1665) $\frac{2}{\pi} = \left[\frac{1\cdot 3\cdot 5\cdots}{2\cdot 4\cdot 6\cdots}\right]^2$ that is related to the *odd-even* number infinite product.

The spin $\frac{n}{2}$ distinguishes between fermions ($n = odd$) and bosons ($n = even$). The elemental charge defined by (1) in [esu] is

$$\mathbf{e} = \pm(\alpha\hbar c)^{1/2} = \pm(\hbar c)^{1/2}\frac{M}{K(2\Phi)}\left[\frac{1\cdot 3\cdot 5\cdots}{2\cdot 4\cdot 6\cdots}\right]^2 \tag{5}$$
$$= \pm\alpha^{1/2}\cdot \mathbf{q_P} = \pm\frac{(\hbar c)^{1/2}}{2}(1\cdot 3\cdot 5\cdots)\mathbb{C}$$

The fermion with half-integral spin $s = \hbar/2$ is based on $\mathbf{e} \cdot \mathbf{e}$ interaction

$$\frac{|\mathbf{e}|^2}{c} = \pm\alpha\hbar = \pm\hbar\frac{\alpha^{1/2}M}{K(2\Phi)}\left[\frac{1\cdot 3\cdot 5\cdots}{2\cdot 4\cdot 6\cdots}\right]^2 \tag{6}$$
$$= \pm\frac{\hbar}{2}(1\cdot 3\cdot 5\cdots)\mathbb{C} \Rightarrow \pm\frac{n}{2}\hbar \quad (n = 1, 3, 5\cdots)$$

(5) and (6) shows that a particle with elementary charge $\mathbf{e}$ defined by $\alpha^{1/2}$ in (1) is a spin-half Fermion. Pauli proposed, "*The spin value 1/2 is discriminated through the possibility of a definite charge density, and the spin values 0 and 1 are discriminated through the possibility of defining a definite energy density.*" [4] This is easily confirmed by the photon $\gamma = \hbar\omega = \hbar c/\lambda = \mathbf{q}_\mathrm{p}^2/\lambda$, which is the spin-1 gauge boson in the electromagnetic field. This is also supported by calculating the proton-electron mass ratio with the fine structure constant [3]

$$\beta_{p/e} = (\underbrace{\alpha^{-3/2}}_{\substack{1836\\ \text{Mag}}} - \underbrace{2\alpha^{1/2}}_{\substack{1526\\ \text{Ele}}} + \underbrace{\Phi^2\alpha^2/\pi}_{\substack{724\\ \text{Geo}}} - \underbrace{\eta\alpha^3}_{\substack{7000\\ \text{EW}}})\ln\pi \tag{7}$$
$$= [\underbrace{(\tfrac{\mathbf{q}_\mathrm{P}}{\mathbf{e}})^3}_{\text{Strong>}} - \underbrace{2\tfrac{\mathbf{e}}{\mathbf{q}_\mathrm{P}}}_{\text{Ele>}} + \underbrace{\tfrac{1}{\pi\phi^2}\tfrac{r_e}{a_0}}_{\text{H-Atom>}} - \underbrace{\tfrac{r_e}{a_0}\tfrac{r_e}{\lambda_c^e}\eta}_{\text{E-W}}]\ln\pi$$
$$= \underline{1836}.\underline{1526}\,\underline{724}\,\underline{7000}$$

where the *Planck charge* is the major contributor for the mass/energy of a proton. Protons and Neutrons are proportional to the cubic $\alpha^{-3/2}$ that relates to three color charged quarks and mainly gluons, where $\alpha^{-1/2} = K\Phi\pi\ln 10 = \mathbf{q}_\mathrm{P}/\mathbf{e} = 11.706237669941$. The gluon is a spin-1 vector boson, like the photon. Therefore, "*the spin values 0 or 1*" indeed involve the energy density of the particle. The mass of Baryons are composed primarily of gluons but not quarks. Theoretically, the Dirac magnetic monopole $g \equiv \frac{\hbar c}{2\mathbf{e}} = \frac{1}{2}\frac{\mathbf{q}_\mathrm{P}}{\alpha^{1/2}}$ is also a boson in a scalar field, but has never been found experimentally. [8–10] From (1)

$$g = \frac{1}{2}\frac{q_P}{\alpha^{1/2}} = \pm\frac{q_P}{2}\left[\frac{(2n)!!}{(2n+1)!!}\right]^2 \frac{2K}{M\phi} \tag{8}$$
$$= \pm\frac{(\hbar c)^{1/2}}{2^2}\frac{K}{M\Phi}\frac{(2\cdot 4\cdot 6\cdots)^2}{(1\cdot 3\cdot 5\cdots)^2} = \pm(\hbar c)^{1/2}(1\cdot 2\cdot 3\cdots)\mathbb{C}$$

It also has [esu] dimension but is not an electric charge, $q_P = (\hbar c)^{1/2}$ in (8) is a Planck charge. Similar to (6)

$$\frac{g^2}{c} = \pm\frac{\hbar}{4\alpha} = \pm\frac{\hbar}{2^2}\frac{K}{M\Phi}\frac{(2\cdot 4\cdot 6\cdots)^2}{(1\cdot 3\cdot 5\cdots)^2}\alpha^{-1/2} \tag{9}$$
$$= \pm\hbar(1\cdot 2\cdot 3\cdots)\mathbb{C} \Rightarrow \pm\frac{n}{2}\hbar \quad (n = 2, 4, 6\cdots)$$

It indeed has an integer spin if it exists. Therefore, $\alpha^{+1/2}$ is related to electric charge and $\alpha^{-1/2}$ is related to magnetism (e.g. $R_{xy} = \pm\frac{Z_0}{2}\alpha_\nu^{-1/2}\alpha^{-1/2}$ in I/FQHE). [2] The magnetic coupling is $\beta_m = \frac{g^2}{\hbar c} = \frac{1}{4\alpha} = \frac{\lambda_c^e}{4r_e} = \frac{\sigma_0}{G_0} = 34.259$. In $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, the strong-electroweak coupling is $\alpha_{swe} = m_{\nu_e}/m_{\nu_\mu} \approx 34.26^{-1}$. [11]

From (5) and (9), the Fermion and Boson can be easily discerned by flipping the *even* and *odd* consecutive sequences (It also flips $\Phi = 1/\phi$ and $\log e = 1/\ln 10$). The linkage between the Spin-Statistics and the *odd/even* numbers by the α math formula is what Pauli was looking for. [4] Therefore, (1) meets the requirement of Pauli. This is another proof of the correctness of (1).

A well known unexplainable experiment by Stern-Gerlach in 1922 [70] showed that the electron spin is a non-selectable intrinsic property. Three layers of magnetic fields were used in an effort to obtain only electrons with "up-right" spin (**Fig. 1**). However, after splitting electron beams from "up-down" and "left-right", the result was not the predicated 'up-right' only beam; the electron spin split again into "up-down". Quantum theory so far can not explain why the spin of electrons refuses to be selected. Some have tried using the uncertainty principle but have thus far failed.

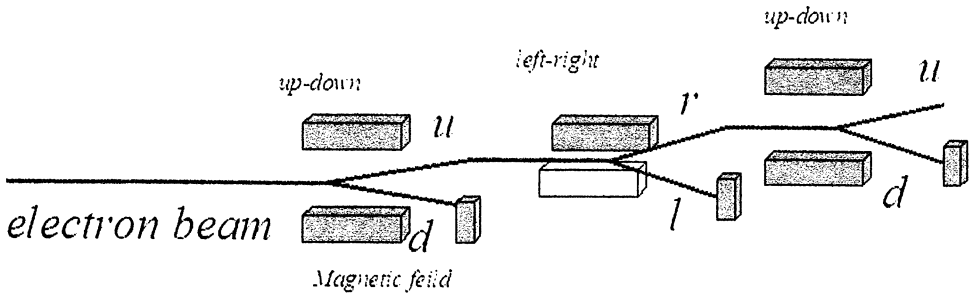


Fig. 1: Stern-Gerlach in 1922, shows that the electron spin is a unselectable intrinsic property. Three layers of magnetic fields were used to try to obtain only electrons with "up-right" spin. However, after splitting the "up-down" and "left-right", the electron beam was not the predicated 'up-right' only beam, the electron spin is split again into "up-down".

(6) shows that the fermion with half-integral spin $s = \hbar/2$ is based on $\mathbf{e}\cdot\mathbf{e}$ interaction $\frac{\mathbf{e}^2}{c} = \pm\alpha\hbar = \pm\frac{\hbar}{2}(1\cdot 3\cdot 5\cdots)\mathbb{C}$. $m_s = \pm\frac{1}{2}$ appears in the Zeeman effect, the hydrogen spectra fine structure, *etc*, [72] where the external magnetic field is equivalent to the electron pair (e.g., I/FQHE $\frac{\pm\mathbf{e}N}{\mathbf{B}} = \frac{\pm\mathbf{e}}{h/\mathbf{e}\nu} = \frac{\pm\mathbf{e}}{h/\nu_i\mathbf{e}} = \frac{\pm\mathbf{e}\mathbf{e}}{h/\nu_i}$). [2] The Schrödinger wavefunction Ψ_{nlm} indeed describes *twins* in a box. [67] The two eigenvalues of $S_\pm = \pm\frac{\hbar}{2}$ are presented as $|s_+ = +\frac{1}{2}\rangle = |\uparrow\rangle = |0\rangle$ and $|s_- = -\frac{1}{2}\rangle = |\downarrow\rangle = |1\rangle$. The electron pair always has an antisymmetrical state, if one is "up" then another must be "down", which must have the opposite spin if two electrons are paired. In 1925, Pauli suggested an empirical rule that *for two electrons residing in the same orbital with identical* n, l, m_l, m_s *must be different and the two electrons will have opposite spins.* By extension, Hund's rule of maximum multiplicity in 1925: *it can be taken that if two or more orbitals of equal energy are available, electrons will occupy them singularily before filling them in pairs.* In 1946, Pauli said that "From the point of view of logic, my report on «Exclusion principle and quantum mechanics» has no conclusion. I believe that it will only be possible to write the conclusion if a theory will be established which will determine the value of the fine-structure constant and will thus explain

the atomistic structure of electricity, which is such an essential quality of all atomic sources of electric fields actually occurring in Nature." [4] The Pauli exclusion principle and Hund's rule are addressed by (1) and (6).

3 Classical Electrodynamics

The electromagnetic field is described as a tensor by the electromagnetic four-potential $A^\mu = (\phi/c, \mathbf{A})$ and $A_\mu = \eta_{\mu\nu}A^\mu = (\phi/c, -\mathbf{A})$, where $\mathbf{A}$ is the vector potential, ϕ is the scalar potential and c is the speed of light. The electric and magnetic fields can be expressed as $\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla\phi$ and $\mathbf{B} = \nabla \times \mathbf{A}$. Then, $F_{\mu\nu} \stackrel{\text{def}}{=} dA_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu$, where

$$F_{\mu\nu} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{bmatrix} \tag{10}$$

and

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{bmatrix} \tag{11}$$

The two matrices with six independent components (the electric field E_x, E_y, E_z and the magnetic field B_x, B_y, B_z) obey: (a) Antisymmetry $F^{\mu\nu} = -F^{\nu\mu}$; (b) the inner product and Lorentz invariance $F_{\mu\nu}F^{\mu\nu} = 2(\mathbf{B}^2 - \mathbf{E}^2)$; (c) Pseudo-scalar invariant $\frac{1}{2}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta} = -4(\mathbf{B}\cdot\mathbf{E})$; and (d) Determinant $\det(F) = (\mathbf{B}\cdot\mathbf{E})^2$. In the field tensor, the Maxwell equations are $\partial_\mu F^{\mu\nu} = j_\nu$ and $\partial_\mu \widetilde{F}^{\mu\nu} = 0$. The Hamilton is

$$\mathcal{H}_{EM} = \frac{1}{2}|F|^2 = \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) \tag{12}$$

and Lagrangian is

$$\mathcal{L}_{EM} = \frac{-1}{4}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) \tag{13}$$

However, $g = 1$ yielded by classical electrodynamics is rejected by the experiment. The semiclassical Bohr model of the hydrogen atom gives the electron spin $g_B = 1$ and the electron angular momentum as $\hbar$.

4 Theory of Quantum and Quantum Field

Modern quantum mechanics started with Heisenberg's discovery of Matrix mechanics in 1925. In 1926, Schrödinger's discovery of the math equivalent Wave mechanics allowed easier understanding of quantum mechanics. However, Schrödinger's equation only considered the static electric potential $\mathbf{e}^2/r$ and ignored the magnetic field. Pauli's exclusive principle involved the Fermion, such as electron's spin $s = \pm\frac{1}{2}$ can not be derived by nether Heisenberg nor Schrödinger. In 1928, Dirac proposed a relativistic field equation for the electrons with two extra terms of electric and magnetic fields $\frac{\mathbf{e}\hbar}{c}(\sigma, H) + i\frac{\mathbf{e}\hbar}{c}\rho_1(\sigma, E)$. [1]

$$[-i\gamma^\mu(\partial_\mu - i\mathbf{e}A_\mu) + m]\psi = 0 \tag{14}$$

It naturally yields $s = \pm\frac{1}{2}$ and $g_D = 2$ with the electron angular momentum as $\hbar/2$, same as the Bohr model, but still does not match the experimental g-factor of electron which is slightly larger than 2. In 1933, the dimensionless "Pauli moment" $((\frac{g}{2} - 1) \sim a_e$ in the same order of $\alpha)$ was introduced after the "Dirac moment" $(\frac{g_D}{2} = 1)$ in the Bohr magneton unit $\mu_B = \frac{\mathbf{e}}{m_e}\frac{\hbar}{2}$ [12]. Pauli modified Dirac's equation (14) to

$$[\underbrace{-i\gamma^\mu(\partial_\mu - i\mathbf{e}A_\mu) + m}_{\text{Dirac}} - \underbrace{(\tfrac{g}{2} - 1)\tfrac{i\mathbf{e}}{2m}F_{\mu\nu}S^{\mu\nu}}_{\text{Pauli}}]\psi = 0 \tag{15}$$

The Dirac free rest-electron $(v = 0)$ was retreated by Pauli as the scattering moving electron $(v > 0)$. This defines

$$g_e = 2[1 + (\frac{g_e}{2} - 1)] = 2(1 + a_e) \tag{16}$$

i.e., Pauli introduced the electron anomaly $a_e = \frac{g-2}{2}$ that is a just little bit larger than zero.

5 QFE and S-matrix

It looks hopeless to solve the Quantum Field equation (15), where the Lagrangian density is given by QED

$$\mathcal{L}_{QED} = \overline{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{17}$$

Since the matrix mechanics and the wave mechanics are mathematically equivalent, from 1934 to 1946, Heisenberg proposed a S-matrix $S = e^{i\eta}$ with property of $SS^\dagger = 1$ to make $\psi_{\text{out}} = S\psi_{\text{in}}$, which involves the orthogonal Hermite polynomials $H_n^{(n)}(x) = 2^n n!$. [13] He said that "*Only in a unified theory of matter and light fields, which would assign a definite value to the Sommerfeld constant* $\mathbf{e}^2/\hbar c$, *would a consistent combination of the requirements of quantum theory and those of correspondence with the intuitive field theory be possible.*" [13] The S-matrix is a scattering matrix representation of the Poincaré group. It was followed by Wheeler in 1937, [14] later developed by Feynman's path-integral for QED, [15] and the Gell-Mann matrix for QCD. [16]

The Heisenberg S-matrix in 1934 is an ingenious concept, and the path-integral and Green's functions are very complicated. Heisenberg thought that "*It is then necessary only to also develop the matrix S that is appropriate for the inhomogeneity in the wave equation in* ***powers of charge****, and to consider the individual terms in the development that are produced by the perturbation process in turn at the successive degrees of approximation.*" [13] Therefore, "*the basic concept is easily illustrated since the S-matrix is expanded in* ***a power series in the electric charge****.*" [17, 18] In other words,

$$S(k) = S_0(k) + S_1(k)\mathbf{e}^1 + S_2(k)\mathbf{e}^2 + S_3(k)\mathbf{e}^3 + S_4(k)\mathbf{e}^4 + \cdots \tag{18}$$

It naturally links to the fine structure constant defined by Sommerfeld in 1916,

$$\alpha = \frac{\mathbf{e}^2}{\hbar c} = 2\pi\frac{\mathbf{e}^2/\lambda}{h\nu} = 2\pi\frac{E_\phi}{E_e} \tag{19}$$

which is proportional to the photon and electron energy ratio (E_ϕ/E_e). We can split the Heisenberg infinite series (18) into two series related to the *odd* and *even* power of electrons $\mathbf{e}^n \sim \alpha^{n/2}$

$$\begin{aligned} S &= \sum_{n=0}^{\infty} S_n(k)\mathbf{e}^n = \sum_{n=0}^{\infty} c_n \alpha^{n/2} \\ &= c_0\alpha^0 + c_1\alpha^{1/2} + c_2\alpha^1 + c_3\alpha^{3/2} + c_4\alpha^2 + \cdots \\ &\qquad \cdots + c_{n+1/2}\alpha^{n+1/2} + c_{2n}\alpha^n + \cdots \\ &= (c_0\alpha^0 + c_2\alpha^1 + c_4\alpha^2 \cdots + c_{2n}\alpha^n + \cdots) + \\ &\qquad +(c_1\alpha^{1/2} + c_3\alpha^{3/2} \cdots + c_{2n+1}\alpha^{n+1/2} + \cdots) \\ &= \sum_{n=0}^{\infty} c_{even}\alpha^{even/2} + \sum_{n=0}^{\infty} c_{odd}\alpha^{odd/2} \\ &= \sum_{n=0}^{\infty} c_{2n}\alpha^n + \sum_{n=0}^{\infty} c_{2n+1}\alpha^{n+1/2} \end{aligned} \tag{20}$$

where $c_0\alpha^0 = 1$ is for the leptons with one elementary charge, $\alpha^{1/2} = \frac{\mathbf{e}}{(\hbar c)^{1/2}} \approx \frac{1}{11.7}$ and $\alpha = \frac{\mathbf{e}^2}{\hbar c} \approx \frac{1}{137}$. According to (19), the scattering matrices (18) and (20) express m-electrons interacting with the electromagnetic field as they exchange n-photons. Then, the complicated electromagnetic field equation is simplified into Einstein's photon, Milliken's electron, and bridged by Sommerfeld's fine-structure constant. When Pauli excitedly worked on "effects proportional to powers of the fine structure constant $\mathbf{e}^2/\hbar c$", [12] however, Heisenberg wrote to Pauli: "this goal cannot be achieved before one is able to fix the value of $\mathbf{e}^2/\hbar c$". [19] Therefore, (20) is still divergent and uncountable. Since that time, Pauli and many others became crazy searching for the fine structure constant math formula.

6 QED Renormalization

6.1 Work of Schwinger, Feynman and Dyson

In order to calculate the Quantum Field equation or the equivalent S-matrix, Feynman worked on a relativistic "*cut-off*" for the divergence

problem, just as Einstein derived $E = mc^2$ through a "*cut-off*" of the Lorentz transform expansion. He developed the S-matrix as $e^{iS/\hbar}$, approved the equivalence of the path-integral and wave-function, invented the simple Feynman diagram with the process probability $P \propto \alpha^n$. [20,21] The breakthrough was made in 1947, considering the "electronic mass" in $\mu = Gm$, Schwinger ingenuously discovered "$e\delta\mu/\mu = (1/2\pi)\mathrm{e}^2/hc = 0.001162$" where h is $\hbar$ [22] i.e.,

$$\frac{\delta\mu}{\mu} = \frac{\alpha}{2\pi} = \frac{1}{2\pi}\alpha = \frac{r_e}{\lambda_c^e} = \frac{E_e}{E_\phi} \approx a_e \tag{21}$$

and

$$g_e \simeq 1 + \frac{\alpha}{2\pi} = 1.001161 \tag{22}$$

His idea got a quick response because it fit the measurement. Schwinger's first paper is only three-pages long, without any theoretical derivation, just as Heisenberg did with the matrix calculation (in fact, Heisenberg did not even realize his calculation was a matrix, later recognized by Born). Getting the numbers correct first is how the quantum pioneers started their job. (21) was originally proposed for the free electron. [23] Feynman later confirmed that $\frac{\alpha}{2\pi}$ also works for the bound electron in the hydrogen atom, where $v_e = \alpha c$ as Bohr's model. [24] This may be acceptable for the first approximation, e.g., the electron mass correction for the hydrogen atom is $\frac{\alpha}{2\pi}(1-\alpha^2/2) = 0.00116140 \times 0.99997 \simeq 0.001161378$. Since then, Schwinger's $a_e \approx \frac{\alpha}{2\pi}$ became the cornerstone of the Lepton anomaly calculation.

Based on Schwinger's $a_e \approx \frac{\alpha}{2\pi}$, and Feynman's "cut-off" $e^{iS/\hbar}$ by "sweeping the difficulties under the rug", Dyson worked on the Heisenberg-Feynman S-matrix in 1949. He believed electron mass and charge must be modified by the electromagnetic field [25,26]

$$\begin{aligned} \delta m &= m(A_1\alpha + A_2\alpha^2 + \cdots) \\ \mathrm{e}' &= \mathrm{e}(1 + \underline{c_2\alpha + c_4\alpha^2 + \cdots}) \end{aligned} \tag{23}$$

where e is the observable electric charge, and $\mathrm{e}^2/\hbar c = \alpha$. Unlike Schwinger who worked on the "mass", Dyson only considered the "true" electron

charge $\mathbf{e}'$ interactions, i.e., the α^n terms (n is integer) in the Heisenberg S-matrix (20). Dyson not only "cut-half" (i.e., ignored those terms of $\sum_{n=0}^{\infty} c_{2n+1}\alpha^{n+1/2}$ in (20)), but also further "cut-off" the infinite series to $\sum_{n=0}^{N} c_{2n}\alpha^n$. According to Dyson, if we assume $\delta m = 0$ in (23), then

$$\mu_e = \frac{g_e}{2}\frac{\mathbf{e}\cdot s}{m_e} = [\mathbf{e}\cdot(1+\underline{c_2\alpha + c_4\alpha^2 + \cdots + c_{2n}\alpha^n})]\cdot\frac{s}{m_e} \tag{24}$$

i.e., the electron anomaly is

$$a_e = c_2\alpha + c_4\alpha^2 + \cdots + c_{2n}\alpha^n \tag{25}$$

while $c_2 = \frac{1}{2\pi}$ was already found by Schwinger. However, if $m' = m + \delta m$ in (23), then

$$a_e = \frac{c_2\alpha + c_4\alpha^2 + \cdots}{A_1\alpha + A_2\alpha^2 + \cdots} \tag{26}$$

which is ignored by QED. We have shown the electron in hydrogen atom has the mass defect $\triangle m_e = m_e\alpha^2/2 = hR_\infty/c = \varepsilon_0\mu_0 \mathrm{Ry}$. [3] However, the mass difference will not affect the anomaly much. For example, the muon is much heavier than the electron, but their anomaly is almost the same; while a proton and neutron have a similar mass, but their g-2 values have a big difference based on electric charges 1 and 0.

The interaction of m-electron H^e and n-photons H^I are given by the Dyson series of multiple-integrals

$$\begin{aligned} U(\infty) &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(\frac{-i}{\hbar c})^{m+n}\frac{1}{m!n!}\int_{-\infty}^{\infty}dx_1\cdots\times\int_{-\infty}^{\infty}dx_{m+n} \\ &\quad\times P[H^e(x_1),\cdots,H^e(x_m)\times H^I(x_{m+1}),\cdots,H^I(x_{m+n})] \end{aligned} \tag{27}$$

where $U(\infty) = S(\infty)$. It was later named the Feynman-Dyson rules in parametric space. [27]

6.2 The c_4 calculation in 1950s

Following Dyson, Karplus and Krell did the initial calculation for $c_4\alpha^2$, which involves **1**-electron and **4**-photons. According to (27), $m + n = 1 + 4 = 5$ and

$$U(4) = (\frac{-i}{\hbar c})^5 \frac{1}{4!} \int_{-\infty}^{\infty} dx_0 dx_1 dx_2 dx_3 dx_4 \times \quad (28)$$
$$\times P(H^e(x_0), H^I(x_1), H^I(x_2), H^I(x_3), H^I(x_4))$$

They studied 18 Feynman diagrams but only calculated 5 of them. Unfortunately, the calculation was beset with low-level mistakes, yielding a number of 2.973, close to the proton g-2 value, but not to the electron g-2 experiments [28]

$$-2.973(\frac{\alpha}{\pi})^2 = [-\frac{2687}{288} + \frac{125}{36}\pi^2 + 28\zeta(3) - 9\pi^2 \ln 2](\frac{\alpha}{\pi})^2 \quad (29)$$

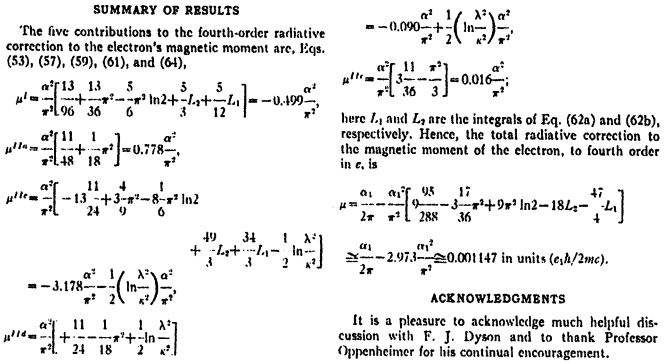
In 1957, Peterman (with Bohr) and Sommerfield (with Schwinger) made simple arithmetic corrections using different mathematical methods independently, both getting

$$-0.328(\frac{\alpha}{\pi})^2 = [\frac{197}{144} + \frac{1}{12}\pi^2 + \frac{3}{4}\zeta(3) - \frac{1}{2}\pi^2 \ln 2](\frac{\alpha}{\pi})^2 \quad (30)$$

again, following the electron g-2 experimental result. [29, 30] However, they didn't change the physical model of Karplus and Krell (and still only calculated 5 Feynman diagrams). The difference between (29) and (30) is only at the numbers in μ_I and μ_{IIc} (**Fig. 2**). In 1958, Peterman further reported $[-0.39 \pm 0.11](\frac{\alpha}{\pi})^2$ to better fit with the experimental data. Since then, all c_{2n} calculations must fit to the experiments (an unspoken rule). QED has claimed to match with experiments for over 50 years, simply by changing the calculation result to follow the experiments.

In 1961, Feynman discussed $g_e = 2[1+\frac{\alpha}{2\pi}-0.328(\frac{\alpha}{\pi})^2] = 2(1.00115961)$ as equation (4): "*To make my view clearer, consider, for example, the anomalous electron moment given in (4). We have no physical picture by which we can easily see that the correction is roughly* $\frac{\alpha}{2\pi}$, *in fact, we do not even know why the sign is positive (other than by computing it). In another field we would not be content with the calculation of the second order term to three significant figures without enough understanding*

to get a rational estimate of the order of magnitude of the third. ***We have been computing terms like a blind man exploring a new room, but soon we must develop some concept of this room as a whole, and to have some general idea of what is contained in it. As a specific challenge, is there any method of computing the anomalous moment of the electron*** *which, on first rough approximation, gives a fair approximation to the α term and a crude one to α^2; and when improved, increases the accuracy of the α^2 term, yielding a rough estimate to α^3 and beyond."* [24] His "*whole*" view is still not achieved even today.



SUMMARY OF RESULTS

The five contributions to the fourth-order radiative correction to the electron's magnetic moment are, Eqs. (53), (57), (59), (61), and (64),

$$\mu^{I}=\frac{\alpha^2}{\pi^2}\left[\frac{13}{96}+\frac{13}{36}\pi^2-\frac{5}{6}\pi^2\ln 2+\frac{5}{3}L_2+\frac{5}{12}L_1\right]=-0.499\frac{\alpha^2}{\pi^2},$$

$$\mu^{IIa}=\frac{\alpha^2}{\pi^2}\left[\frac{11}{48}+\frac{1}{18}\pi^2\right]=0.778\frac{\alpha^2}{\pi^2},$$

$$\mu^{IIc}=\frac{\alpha^2}{\pi^2}\left[-13\frac{11}{24}+3\frac{4}{9}\pi^2-8\frac{1}{6}\pi^2\ln 2+\frac{49}{3}L_2+\frac{34}{3}L_1-\frac{1}{2}\ln\frac{\lambda^2}{\kappa^2}\right]$$

$$=-3.178\frac{\alpha^2}{\pi^2}-\frac{1}{2}\left(\ln\frac{\lambda^2}{\kappa^2}\right)\frac{\alpha^2}{\pi^2},$$

$$\mu^{IId}=\frac{\alpha^2}{\pi^2}\left[\frac{11}{24}-\frac{1}{18}\pi^2+\frac{1}{2}\ln\frac{\lambda^2}{\kappa^2}\right]$$

$$=-0.090\frac{\alpha^2}{\pi^2}+\frac{1}{2}\left(\ln\frac{\lambda^2}{\kappa^2}\right)\frac{\alpha^2}{\pi^2},$$

$$\mu^{IIe}=\frac{\alpha^2}{\pi^2}\left[3\frac{11}{36}-\frac{\pi^2}{3}\right]=0.016\frac{\alpha^2}{\pi^2};$$

here L_1 and L_2 are the integrals of Eq. (62a) and (62b), respectively. Hence, the total radiative correction to the magnetic moment of the electron, to fourth order in e, is

$$\mu=\frac{\alpha_1}{2\pi}-\frac{\alpha_1^2}{\pi^2}\left[9\frac{95}{288}-3\frac{17}{36}\pi^2+9\pi^2\ln 2-18L_2-\frac{47}{4}L_1\right]$$

$$\cong\frac{\alpha_1}{2\pi}-2.973\frac{\alpha_1^2}{\pi^2}\cong 0.001147 \text{ in units } (e_1\hbar/2mc).$$

ACKNOWLEDGMENTS

It is a pleasure to acknowledge much helpful discussion with F. J. Dyson and to thank Professor Oppenheimer for his continual encouragement.

(a)

$$\mu_{\rm I} = \frac{\alpha^2}{\pi^2}\left(\frac{1}{6}+\frac{13}{36}\pi^2+\frac{5}{4}\zeta(3)-\frac{5}{6}\pi^2\,\mathrm{Log}\,2\right)=-0.467\,\frac{\alpha^2}{\pi^2}. \quad \bigstar(1)$$

$$\mu_{\rm IIa} = \frac{\alpha^2}{\pi^2}\left(\frac{11}{48}+\frac{\pi^2}{18}\right)=0.778\,\frac{\alpha^2}{\pi^2}. \quad (2)$$

$$\mu_{\rm IIc} = \frac{\alpha^2}{\pi^2}\left(-\frac{67}{24}+\frac{\pi^2}{18}-\frac{1}{2}\zeta(3)+\frac{1}{3}\pi^2\,\mathrm{Log}\,2-\frac{1}{2}\,\mathrm{Log}\,\frac{\lambda^2}{m^2}\right)=$$
$$-0.564\,\frac{\alpha^2}{\pi^2}-\frac{1}{2}\,\frac{\alpha^2}{\pi^2}\,\mathrm{Log}\,\frac{\lambda^2}{m^2}. \quad \bigstar(3)$$

$$\mu_{\rm IId} = \frac{\alpha^2}{\pi^2}\left(\frac{11}{24}-\frac{\pi^2}{18}+\frac{1}{2}\,\mathrm{Log}\,\frac{\lambda^2}{m^2}\right)=-0.090\,\frac{\alpha^2}{\pi^2}+\frac{1}{2}\,\frac{\alpha^2}{\pi^2}\,\mathrm{Log}\,\frac{\lambda^2}{m^2}. \quad (4)$$

$$\mu_{\rm IIe} = \frac{\alpha^2}{\pi^2}\left(\frac{119}{36}-\frac{\pi^2}{3}\right)=0.016\,\frac{\alpha^2}{\pi^2}. \quad (5)$$

$$\mu^{(4)}_{\rm total} = \frac{\alpha^2}{\pi^2}\left(\frac{197}{144}+\frac{\pi^2}{12}+\frac{3}{4}\zeta(3)-\frac{1}{2}\pi^2\,\mathrm{Log}\,2\right)=-0.328\,\frac{\alpha^2}{\pi^2}. \quad (6)$$

(b)

Fig. 2: The c_4 calculation by Karplus in 1950 (a) and Peterman in 1957 (b). The difference is μ_I ★(1) and μ_{IIc} ★(3)

6.3 The Numerical and Analytical c_6

In the 1970s, Cvitanovic joined the QED calculations after Kinoshita challenged him with some easy path-integrals. He made several contributions: (1) setting up a supercomputer numerical fitting program, later developed by Lepage as "the iterative-adaptive Monte Carlo integration routine VEGAS", which is now improved by Kinoshita and others; (2) got a numeral $c_6 = 1.183 \pm 0.011$, later corrected as $c_6 = 1.181259(40)$; (3) developed the graphic theory for the set as c_2, c_4, c_6, c_8, $c_{10} \in 1, 7, 72, 891, 12672$ Feynman diagrams, although the real calculations only involved $c_2, c_4, c_6, \ldots \in 1, 5, 50, \ldots$. He published four papers with Kinoshita before 1974. [31] In 1977 however, he asked "Why is $\frac{\alpha}{\pi}$ and not $\frac{\alpha}{4\pi}$ or something else the expansion parameter?" After discussion with Feynman on looking for a "*whole*" picture, he proposed a non-perturbation gauge-invariance formula of the electron anomaly $a_e = \frac{\alpha}{2\pi}[1 - (\frac{\alpha}{\pi})^2]^{-2}$. [32, 33] At the same time, Landau was also looking for the "*whole*" picture and tested the diffraction related $a_e = g(\phi) = \frac{\alpha}{2\pi}\frac{\phi}{\sin\phi}$ in 1982. [34]

In 1996, Laporta and Remidd reported an analytic c_6, [35]

$$\begin{aligned} c_6 &= \frac{83}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5) + \frac{100}{3}[(a_4 + \frac{1}{24}\ln^4 2) - \frac{1}{24}\pi^2\ln^2 2] \quad (31) \\ &\quad -\frac{239}{2160}\pi^4 + \frac{139}{18}\zeta(3) - \frac{298}{9}\pi^2\ln 2 + \frac{17101}{810}\pi^2 + \frac{28259}{5184} \\ &= 1.181241456\ldots \end{aligned}$$

where $a_4 = \sum_{n=1}^{\infty} 2^{-n} n^{-4}$. (31) involved 79 Feynman diagrams, therefore, QED claimed path-integral calculations using 1, 7, 72, 891, 12672 Feynman diagrams are not true. It is noticed that the numeral and analytic results give $(c_6^N/c_6^A) - 1 = \frac{1.181259(40)}{1.181241456} - 1 = 1.485 \times 10^{-5}$, not very good.

The original QED calculated $a_e = g_e/2 - 1$ from α; starting in 1996, Kinoshita *reversely* calculated the α value from the $g_e/2$ experimental data, and continually worked on the numeral calculation of c_8 (2007) and c_{10} (2010) (his mistakes were corrected by Aoyama in 2012). [36–38] Principally, using one equation to solve many unknowns $(\alpha, c_2, c_4, c_6, c_8, c_{10} \cdots)$ is impossible.

6.4 Numeral c_8, c_{10} and Discussion

According to the principle of Heisenberg's S-matrix, one must first have the α value, before calculating the anomaly a_e. However, the current value $\alpha = \mathbf{e}^2/\hbar c$ is *reversely* calculated by the adaptive-iterative Monte-Carlo integration routine VEGAS, using the *best measured* $g_e/2$ from a single-electron in a magnetic Penning trap. After 20 years of hard work, the Gabrielse group at Harvard can measure a *single* free electron anomaly in the quantum cyclotron at 5.36K in the Penning trap [39–41]

$$a_e = \frac{g}{2} - 1 \cong \frac{\overline{\nu}_a - (\overline{\nu}_z)^2/(2\overline{f}_c)}{\overline{f}_c + \frac{3}{2}\delta + -(\overline{\nu}_z)^2/(2\overline{f}_c)} + \frac{\triangle g_{cav}}{2} \tag{32}$$

They got $a_e^{\text{HV08}} = \underline{0.00115965218073}(28)$. The QED α calculation, based on the Harvard group measurement of $g_e/2$ and a_e, reached 0.70 ppb in 2006. In 2008, $\alpha^{-1}_{\text{HV08}} = 137.035999084\,(51)$, $u_r = 0.37$ ppb, which is matched by the BO-Interferometer experiments $\alpha^{-1}_{\text{BO10}} = 137.035999037\,(91)$, $u_r = 0.66$ ppb in 2010. [42] After 2006, however, CODATA started using α to calibrate all other physical constants. The BO-Interferometer calculation used CODATA values in formula $\alpha^2 = \frac{2R_\infty}{c}\frac{A_r(X)}{A_r(\mathrm{e})}\frac{h}{m_X}$, will retrieve the similar CODATA recommended α value. One should not be fooled by those new reports of testing QED. [42] It is the same circular logic of being surprised at discovering eggs during an Easter egg hunt.

Using (1), let $H = \frac{2}{9}\pi(e + e^{-1})^2$ in $K(J,H) = 1 + \frac{1^H}{J+}\frac{3^H}{J+}\frac{5^H}{J+}\frac{7^H}{J+}\cdots$, yielding $\alpha^{-1}_{6.64828} = 137.03599907395$ to match the CODATA-2010 $\alpha^{-1}_{2010} = 137.035999074(44)$; let $H = 6+\gamma = 6.57721566$ where $\gamma = 0.57721566490$ is the Euler–Mascheroni constant, then $\alpha^{-1}_{6.57721566} = \underline{137.035999173895}$ to match the latest QED calculation $\alpha^{-1}_{2012} = \underline{137.035999173}\,(35)$. [2] (1) can easily fit with any new incoming data, however, the so-called *most reliable* QED calculations have *perpetually* changed in the past 50 years. [43]

The current calculation of the electron anomaly $a_e = (|g_e| - 2)/2$ is up to x^5 ($x = \alpha/\pi = 1/430.511$) [44]

$$\begin{aligned} a_e^{QED} &= c_2(\frac{\alpha}{\pi}) + c_4(\frac{\alpha}{\pi})^2 + c_6(\frac{\alpha}{\pi})^3 + c_8(\frac{\alpha}{\pi})^4 + c_{10}(\frac{\alpha}{\pi})^5 \\ &\quad + \cdots + a_{\mu\tau} + a_{\text{hadronic}} + a_{\text{weak}} \end{aligned} \tag{33}$$

where $c_2 = 0.5$, $c_4 = -0.328478965$, $c_6 = 1.181241456$, $c_8 = -1.91097(20)$ and $c_{10} = 9.16\,(58)$ are built by Schwinger-Sommerfield-Remiddi-Aoyama-Aoyama. [22,30,35,44] The calculation up to c_8 yields two real and two imagery solutions (**4** in **Fig. 3**). Using (33), only the *odd*-powers (*1, 3, 5, ..., 2n+1*) with the alternate sign can have *one* real plus $2n$ imagery solutions for $(\frac{\alpha}{\pi})$ (**Fig. 3**), which needs $c_{10} > 0.755$ for $(\alpha/\pi)^5$ (12672 Feynman diagrams). There is no limitation for the α value in **Fig. 3**, therefore, the QED reported α^{-1} value is always changing.

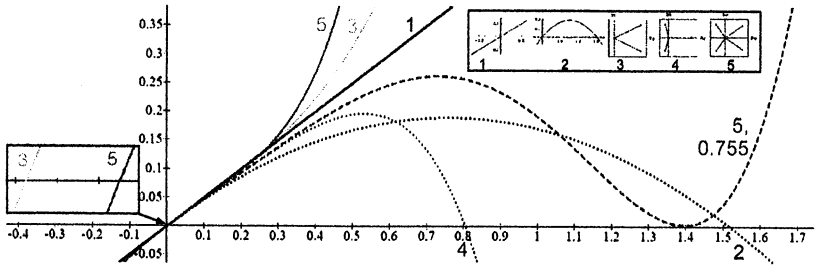


Fig. 3: Perturbation QED α calculation by (33) with multiple solutions. Enlargement shows that there is no limitation.

In (33), there are μ, τ and hadronic mass modifications, those particles do not exist in the single free-electron Penning trap. Till now, QED calculation has only yielded a single number, based on many uncertain numbers, no formulas, a real numerical alchemy, no difference with those guessed α math formulas, but even more difficult to make a judgment. Sadly, the physical constants of CODATA-2010 are based on this number. However, CODATA also reported $\alpha^{-1} = 137.0360000(11)$, $\alpha^{-1} = 137.0360005(20)$ and $\alpha^{-1} = 137.0360018(80)$ in 2012. [75]

Principally, the Feynman diagrams are for $e^2, e^4, \cdots, e^{2n}$ (i.e., $\alpha^n = (\frac{e^2}{\hbar c})^n = (\frac{1}{137})^n$) but not for $\frac{e^2}{\pi}, \frac{e^4}{\pi^2}, \cdots, \frac{e^{2n}}{\pi^n}$ (i.e., $(\frac{\alpha}{\pi})^n = (\frac{2e^2}{hc})^n = (\frac{e}{2\pi})^{2n} = (\frac{1}{430})^n$). [21,39] Schwinger originally proposed $a_e = (\frac{1}{2\pi})\alpha \propto \frac{1}{137}$ but not $(\frac{1}{2})(\frac{\alpha}{\pi}) \propto \frac{1}{430}$. [22] The base of the Dyson series from $\alpha \approx \frac{1}{137}$ switched to $\frac{\alpha}{\pi} \approx \frac{1}{430.5}$ during c_4 calculation. [30] Although there are many integrals that can yield $\frac{1}{2}$, the Feynman path-integral in QED yields 1 (**Fig. 4**), so, Schwinger's ingenious $\frac{\alpha}{2\pi}$ should be $c_2 = \frac{1}{2\pi}$ but not $c_2 = \frac{1}{2}$ in (33).

Neither the ultraviolet nor the infrared divergence affects $F_2(q^2)$, so we just set $q = 0$:

$$F_2(0) = \frac{\alpha}{2\pi}\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\left[\frac{2m^2z(1-z)}{m^2(1-z)^2}\right] + O(\alpha^2) =$$
$$= \frac{\alpha}{2\pi}\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\frac{2z}{1-z} + O(\alpha^2) =$$
$$= \frac{\alpha}{2\pi}\int_0^1 dy\int_0^1 dz\,\theta(1-(1-y-z))\theta((1-y-z)-0)\frac{2z}{1-z} + O(\alpha^2) =$$
$$= \frac{\alpha}{2\pi}\int_0^1 dy\int_0^1 dz\,\theta(y+z)\theta(1-y-z)\frac{2z}{1-z} + O(\alpha^2) =$$
$$= \frac{\alpha}{2\pi}\int_0^1 dy\int_0^1 dz\,\theta(1-y-z)\frac{2z}{1-z} + O(\alpha^2) =$$
$$= \frac{\alpha}{2\pi}\int_0^1 dz\int_0^{1-z} dy\frac{2z}{1-z} + O(\alpha^2) =$$
$$= \frac{\alpha}{2\pi}\int_0^1 dz(1-z)\frac{2z}{1-z} + O(\alpha^2) =$$
$$= \frac{\alpha}{2\pi}\int_0^1 dz2z + O(\alpha^2) =$$
$$= \frac{\alpha}{2\pi} + O(\alpha^2)$$

Fig. 4: Feynman path-integral yields 1 (from QED class at http://theoretical-physics.net/dev/src/quantum/qed.html (2011)

After talking with Sommerfield, Schwinger took $e/\pi^{1/2}$ as the non-interacting scalar bosons of mass in one-dimensional space, where the electric charge e would only have a dimension of mass. [23] This raised another problem, since mass in $\mu_e = \frac{g_e}{2}\frac{e\cdot s}{m_e} = (1+c_2\frac{\alpha}{\pi})\frac{e\cdot s}{m_e}$ is a divisor, it does not fit. Kinoshita said: "The coefficients of $(\frac{\alpha}{\pi})^n$ are mass-dependent" and "the factor $(\frac{\alpha}{\pi})^n$ is suppressed for simplicity." [37] According to Sommerfeld, $\frac{\alpha}{2\pi} = E_e/E_\phi$ is the energy ratio of interactions between the electron and the photon, where $E_e = \frac{e_1 e_2}{\lambda_c}$ is the energy needed to bring two electrons from infinity to a distance of λ_c against their electrostatic repulsion, and $E_\phi = h\nu = \frac{hc}{\lambda_c}$ is the energy of a single photon with a wavelength λ_c. In the Penning trap, there are no muon and tau particles, the mass correlation in (33) is not agreed on the experimental calculation as (32). The Schwinger limit $E_S = \frac{m_e^2c^3}{e\hbar} \simeq 1.3\times10^{18}$ [V/m] of the non-linear correction for QED can not be reached in the Penning trap. [69]

Although Kinoshita's calculations were erroneous (as he said: "Everyone makes mistakes - including Feynman"), [37] Dyson still wrote to Gabrielse in 2006: "*we thought of QED in 1949 as a temporary and*

jerry-built structure, with mathematical inconsistencies and renormalized infinities swept under the rug. We did not expect it to last more than ten years before some more solidly built theory would replace it..." He was surprised the "*ramshackle structure still stands. The theorists ... have kept pace with your experiment, pushing their calculations to higher accuracy than we ever imagined.*" [45] It clearly shows that the α value produced by QED does not have a self-checking capability, and is dependent on the experimental g-2 measurements. The current α obtained from g-2 is an experimentally dependent numerical fitting result, not an analytic formula. It is hard to believe that the fine structure constant α is principally based on the electron anomaly a_e in the strong magnetic field. QED could not answer Pauli's simple question: "Why 137?" Feynman even called the "*renormalization*" as "*a dippy process*", which "*is not mathematically legitimate.*" "*It's one of the* greatest *damn mysteries of physics*: a magic number *that comes to us with no understanding by man.*" [46]

7 Exploring *whole* a_e Formula

Feynman thought that one must look at the ***whole*** picture, and try "***any method of computing the anomalous moment of the electron***" [46] Following Feynman's advice and Schwinger's electron mass modification $\frac{\delta\mu}{\mu} \simeq \frac{\alpha}{2\pi}$, Cvitanovic proposed $a_e = \frac{\alpha}{2\pi}[1 - (\frac{\alpha}{\pi})^2]^{-2}$, [32] and Landau tested $a_e = g(\phi) = \frac{\alpha}{2\pi}\frac{\phi}{\sin\phi}$. [34] In order to get a_e as ***whole***
(1) One must first obtain the α value as Pauli and Heisenberg said;
(2) Dirac's solution of the rest electron $g_D = 2$ and $s = \frac{1}{2}$ must be kept;
(3) Pauli's term for the moving electron ($v_e = \alpha c \gg 0$ as Bohr model) must involve the Lorentz transform for relativistic effects;
(4) Schwinger's discovery $\frac{\delta\mu}{\mu} \simeq \frac{\alpha}{2\pi}$ should be the starting point;
(5) The electron and photon interaction should involve $1/e$ for the B-E and F-D statistics of boson and fermion;
(6) It should fit to the Lagrangian $\mathcal{L}_{EM} = \frac{-1}{2}(\mathbf{E}^2 - \mathbf{B}^2)$ if possible;
(7) The math constants must be basic and physically explainable;
(8) a_e, as an intrinsic property of the electron, must be simple.

Using $\alpha = 0.0072973524999966932$ from (1), we get a simple formula based on Schwinger's term for the "electronic mass" correction, [22] to fit $a_e^{\text{HV08}} = \underline{0.00115965218073}(28)$ and $a_e^{2010} = \underline{0.00115965218076}(27)$

$$\begin{aligned}
a_e &= \frac{\alpha}{2\pi}\cdot\frac{1}{\gamma}\frac{\pi}{e}\left(\frac{64G_E}{31}-\frac{9}{7}\frac{\pi}{11}\right)\\
&= \frac{\alpha}{2\pi}\frac{1}{\gamma}\left(\frac{64\pi}{31}\Gamma(0,1)-\frac{9}{7}\frac{\pi^2}{11e}\right) \qquad (34)\\
&= \alpha\cdot\sqrt{1-\alpha^2}\cdot\left(\frac{32G_E}{31e}-\frac{9}{7}\frac{\pi}{2}\frac{1}{11}\frac{1}{e}\right)\\
&= \frac{\alpha}{\gamma}\left[\frac{1}{1-s^5}\Gamma(0,1)-\frac{9}{7}\frac{\pi}{2}\frac{1}{11}\Gamma(1,1)\right]\\
&= \frac{1}{\gamma}[\underbrace{\alpha\cdot\frac{g_D}{g_D-s^4}\Gamma(0,1)}_{\text{Electric}}-\underbrace{\eta'\cdot\frac{\pi/2}{\Phi^5-\phi^5}\Gamma(1,1)}_{\text{Electroweak}}]\\
&= \gamma^{-1}[\text{A}-\text{B}] = \underline{0.00115965218073}1730
\end{aligned}$$

Again, $\phi-\Phi-e-\pi$ appeared in (34):[1] (a) $\gamma = 1/\sqrt{1-v_e^2/c^2} = 1/\sqrt{1-\alpha^2}$ is a Lorentz factor for the relativistic mass correction $m_e' = \gamma m_e$; (b) $\frac{g_D}{g_D-s^4} = \frac{1}{1-s^5}$ is the g-factor and spin yielded by the Dirac equation ($g_D = 2$ and $s = \frac{1}{2}$); (c) $\frac{\pi}{2} = \frac{2}{1}\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\frac{8}{9}\frac{10}{9}\frac{10}{11}\cdots = \frac{3}{2}\frac{5}{6}\frac{7}{6}\frac{11}{10}\frac{13}{14}\frac{17}{18}\frac{19}{18}\frac{23}{22}\cdots$ is the Wallis-Euler constant; (d) $e^{-1} = \Gamma(1,1) = \sum_{n=0}^{\infty}\frac{(-1)^n}{n!} = \frac{1}{2+}\frac{1}{1+}\frac{1}{2+}\frac{2}{3+}\frac{3}{4+}\frac{4}{5+}\cdots =$ 0.367879... is statistically the sum of the arrangement ratio between photon and electron, related to the incomplete Gamma function; (e) The Euler-Gompertz Constant $G_E = \sum_{n=0}^{\infty}(-1)^n n! = \int_0^{\infty}\frac{e^{-u}}{1+u}du = -e\text{Ei}(-1) = e\Gamma(0,1) = \frac{1}{2-}\frac{1^2}{4-}\frac{2^2}{6-}\frac{3^2}{8-}\cdots = 0.596347362323\ldots$ is also related to the incomplete Gamma function, and statistically, $G_E/e = \Gamma(0,1) = -\text{Ei}(-1) = [\sum_{n=0}^{\infty}(-1)^n n!]\cdot[\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}] = 0.219383934395521$ is [the sum of the arrange-

[1] In (34), the number calculations are given as follow
$a_e^1 = 0.001342237240462(1.231168748022080 - 0.367199141328677) = \underline{0.00115965218073}173$
$a_e^2 = 0.001161378798127(1.422895394296960 - 0.424382090453294) = \underline{0.00115965218073}173$
$a_e^5 = 0.999973373968773(0.001652564544124 - 0.000492881485635) = \underline{0.00115965218073}173$

ment product] times [the sum of the arrangement ratio], between photon and electron; (f) $\alpha = 2\pi E_\phi / E_e$ is the photoelectric coupling constant; and (g) $\eta' = \alpha \cos^{-2}\theta_w = \frac{9}{7}\alpha$ is the electroweak coupling constant related to the weak-mixing angle; (h) $11 = \Phi^5 - \phi^5$. In (34), the relativistic mass correction is $\frac{1}{\gamma} \approx 1$ and the charge modification is $[Ele(\alpha) - EW(\eta')] \approx \frac{\alpha}{2\pi}$. Therefore, the electron magnetic moment is

$$\begin{aligned} \mu_e &= \frac{g_e}{2}\frac{\mathbf{e}\cdot s}{m_e} = (1+a_e)\frac{\mathbf{e}\cdot s}{m_e} = \frac{\mathbf{e}\cdot s}{m_e} + \frac{\mathbf{e}'\cdot s}{m'_e} \\ &= \frac{\mathbf{e}\cdot s}{m_e} + \frac{\mathbf{e}\cdot s}{\underline{\gamma m_e}} \cdot \underbrace{[\alpha \cdot \frac{g_D}{g_D - s^4}\Gamma(0,1)}_{\text{Electric}} - \underbrace{\eta' \cdot \frac{\pi/2}{\Phi^5 - \phi^5}\Gamma(1,1)]}_{\text{Electroweak}} \end{aligned} \tag{35}$$

(35) clearly shows that the electron anomaly is due to (a) the relativistic mass modification γm_e; and (b) the charge modification comes from two terms: the electric term with α, and the electroweak term with $\eta' = \alpha \cos^{-2}\theta_w = \frac{9}{7}\alpha$. Both modifications involve the incomplete Gamma function $\Gamma(0,1)$ and $\Gamma(1,1)$. The incomplete Gamma function is

$$\Gamma(n,\, x) = (n-1)!e^{-x}\sum_{k=0}^{\infty}\frac{x^k}{k!} \tag{36}$$

Therefore, the electron anomaly is a charge with electromagnetic and electroweak coupling, plus a relativistic mass effect for the free electron cyclotron motion ($v_e = \alpha c$) (**Fig. 5**).

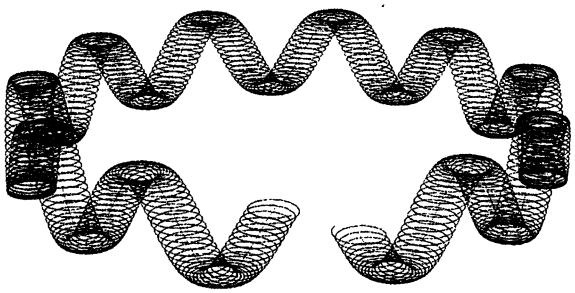


Fig. 5: The single free electron motion in a Penning trap

In the infinite series, infinite product and continued fraction, the electron anomaly (34) is

$$a_e = \frac{\alpha\sqrt{1-\alpha^2}}{\sum_{n=0}^{\infty}\frac{(+1)^n}{n!}} \cdot \left\{ \frac{\frac{1}{2-}\frac{1^2}{4-}\frac{2^2}{6-}\frac{3^3}{8-}\cdots}{1-\prod_{n=1}^{5}\cos(\frac{n\pi}{11})} - \frac{\prod_{n=1}^{\infty}\frac{(2n)^2}{(2n-1)(2n+1)}}{[\cos(\theta_w)\cdot\prod_{n=1}^{5}\tan(\frac{n\pi}{11})]^2} \right\} \quad (37)$$

$$= \alpha\sqrt{1-\alpha^2}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\left\{ \frac{\sum_{n=0}^{\infty}(-1)^n n!}{1-\prod_{n=1}^{5}\cos(\frac{n\pi}{11})} - \frac{\sum_{n=0}^{\infty}\frac{n!}{(2n+1)!!}}{[\cos(\theta_w)\prod_{n=1}^{5}\tan(\frac{n\pi}{11})]^2} \right\}$$

where $\frac{1}{\sqrt{11}} = \prod_{n=1}^{5}\cot(\frac{n\pi}{11})$ and $s^5 = (\frac{1}{2})^5 = \frac{1}{32} = \prod_{n=1}^{5}\cos(\frac{n\pi}{11})$ are the Newton triangular identities. The Newton identities $11 = [\prod_{n=1}^{5}\tan(\frac{n\pi}{11})]^2$ and $s^5 = (\frac{1}{2})^5 = \frac{1}{32} = \prod_{n=1}^{5}\cos(\frac{n\pi}{11})$ are the natural "cut-off".

Muons only have a 2 μs lifetime and are 206 times heavier than electrons. Although it's not pretty, if we multiply $\frac{11(e\pi\ln\pi)^2\zeta(-3)}{4\ln(2\pi)\ln^2(3)}$ to the second term B of [A − B] in (34), one gets the muon anomaly $a_\mu^{exp} =$ 0.00116592091(63)

$$\begin{aligned} a_\mu &= \frac{\alpha}{2\pi}\cdot\frac{1}{\gamma}\frac{\pi}{e}\left(\frac{64G_E}{31} - \underline{\frac{\cancel{11}(e\pi\ln\pi)^2\zeta(-3)}{4\ln(2\pi)\ln^2(3)}\cdot\frac{9}{7}\frac{\pi}{\cancel{11}}}\right) \\ &= \frac{\alpha}{2\pi}\frac{1}{\gamma}\left(\frac{64\pi}{31}\Gamma(0,1) - \underline{\frac{(e\pi\ln\pi)^2\zeta(-3)}{4\ln(2\pi)\ln^2(3)}\cdot\frac{9}{7}\frac{\pi^2}{e}}\right) \quad (38) \\ &= \frac{\alpha}{\gamma}\left[\frac{1}{1-s^5}\Gamma(0,1) - \underline{\frac{(e\pi\ln\pi)^2\zeta(-3)}{4\ln(2\pi)\ln^2(3)}\cdot\frac{9}{7}\frac{\pi}{2}\Gamma(1,1)}\right] \\ &= \frac{1}{\gamma}[\underbrace{\frac{\alpha}{1-s^5}\Gamma(0,1)}_{\text{Electric}} - \underbrace{\frac{\eta'}{5!\ln(2\pi)}\cdot\frac{e^2\ln^2\pi}{\ln^2(3)}\cdot\frac{\pi^3}{2^3}\Gamma(1,1)}_{\text{Electroweak}}] \\ &= \underline{0.001165920910}0287 \end{aligned}$$

where $\zeta(-3) = 1/5! = 1/120$. The physical meanings of (34)~(38) are correct without any doubt, however, [A – B] is a result of data fitting, just like Feynman said "***any method of computing the anomalous moment of the electron***," which may be improved and explained later.

8 Exploring the Perturbative a_e Formula

The QED calculation (33) has no analytic formula, but gives an extremely complicated numeral data sheet with no physical explanation. Using the latest QED calculation $\alpha_{2012}^{-1} = \underline{137.035999173}\,(35)$, someone got a simple math-equivalent equation $1 + (\frac{\alpha}{2\pi}) - (\frac{\alpha}{2\pi\phi\sqrt{2}})^2 + (\frac{\alpha}{2\pi\gamma})^3 - (\frac{\alpha}{2\pi\Phi})^4 = g/2 = 1.001159652180710$ to match with HV2008 g-2 data. It is a numerical fitting with mistakes, obviously. [2] [66]

Following Heisenberg's S-matrix, to test if $\alpha = 0.007297352499996$ from (1) can create the understandable perturbation formula, a quintic function is built in the alternating series, $a_e = \sum_{n=1}^{5} (-1)^{n-1} c_{2n} \alpha^n$,

$$\begin{aligned} a_e &= \sum_{n=1}^{5} (-1)^{n-1} c_{2n} \alpha^n \\ &= \frac{1}{2\pi}\alpha^1 - \frac{1}{3\pi^2}\alpha^2 + \frac{1}{3\pi}\alpha^3 - \frac{1}{\pi^2}\alpha^4 + \frac{\pi - 2}{2}\alpha^5 \\ &= \sum_{n=1}^{4} (-1)^{n-1} c_{2n} \alpha^n + \mathcal{O}_e(\alpha^5) \\ &= \underline{0.00115965216}891 + 1.181 \times 10^{-11} \\ &= \underline{0.00115965218}072 \end{aligned} \tag{39}$$

where $\mathcal{O}_e(\alpha^5) = 1.181 \times 10^{-11}$ is within range of the *Lamb shift* ($\propto \alpha^5 = 2.096 \times 10^{-11}$), [67] and is very sensitive to the magnetic field of the electron cyclotron (~5 Tesla). The magnetic perturbation may be the cause of discrete data in the determination of the electron anomaly. There is no nucleon in the electron cyclotron, *Hyperfine splitting* ($\propto \alpha^4/\beta =$

[2] $g/2 \simeq 1 + (\alpha/2\pi) - (\Phi\alpha/2\pi\sqrt{2})^2 + (\alpha/2\pi\gamma)^3 - (\alpha/2\pi\Phi)^4 = 1.00115965218075$, where $\gamma = 0.57721566490132$, $\alpha^{-1} \simeq 137.035999168$ from $\sin\alpha^{-1} \simeq 504/85\kappa$ and $\kappa = 8.70036625208 \simeq e^2 \sec 32\check{r}$, or from $\alpha^{-1} \simeq 157 - 337\rho/7$ and $1 + \rho^2 \simeq \kappa/e^2$ to match with $\alpha^{-1} \simeq 137.035999173(35)$.

1.544×10^{-12}) may not account for this free-electron. [67] It is also not necessary to worry about the effective mass of the electron. (39) does not go against QED and returns to its principle $a_e = \sum_{n=0}^{n} (-1)^{n-1} c_{2n} (\frac{1}{137})^n$. In (39), Schwinger's $c_2 = \frac{1}{2\pi}$ is kept; [22] $c_4 = \frac{1}{3\pi^2}$ is derived by Kawahara, considering Relativistic Symmetry without the vacuum polarization. [47] The rest of c_{2n} follow the same pattern and naturally yield $c_2/c_6 = \frac{2}{3}$ and $c_8/c_4 = \frac{1}{3}$. [3]

The spin magnetic moment (39) can be expressed as

$$\begin{aligned} a_e(R) &= R - \frac{4}{3}R^2 + \frac{8\pi^2}{3}R^3 - (4\pi)^2 R^4 \\ &\quad + (\frac{\pi}{2} - 1)(2\pi)^5 R^5 \end{aligned} \tag{40}$$

where $R = \frac{\delta\mu}{\mu} = \frac{\alpha}{2\pi} = \frac{r_e}{\lambda_c^e} = \frac{e^2}{hc} = \frac{E_e}{E_\phi}$.

(39) can also be written in logarithmic format

$$\begin{aligned} a_e &= \frac{1}{2\pi}\alpha^1 - \frac{1}{3\pi^2}\alpha^2 + \frac{1}{3\pi}\alpha^3 - \frac{1}{\pi^2}\alpha^4 + \frac{\pi - 2}{2}\alpha^5 \\ &= \ln\left\{ \frac{\exp[\frac{\alpha^3}{2\pi}(\alpha^{-2} + \frac{2}{3})]}{\exp[\frac{\alpha^2}{\pi^2}(\alpha^{+2} + \frac{1}{3})]} \cdot \frac{\exp[E(0)\alpha^5]}{\exp[E(1)\alpha^5]} \right\} \\ &= \ln\left\{ \frac{\exp[\frac{\alpha^3}{2\pi}(\alpha^{-2} + \frac{2}{3})]}{\exp[\frac{\alpha^2}{\pi^2}(\alpha^{+2} + \frac{1}{3})]} \cdot e^{\mathcal{O}_e} \right\} \\ &= \underline{0.00115965218072} \end{aligned} \tag{41}$$

The electron g-factor $g_e/2 = 1 + a_e$ can be presented as

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[3] $c_2 = 1/2\Gamma^2(1/2)$; $c_4 = 1/18\zeta(2)$; $c_6 = 1/3\Gamma^2(1/2)$; $c_8 = 1/6\zeta(2)$; $c_{10} = \frac{\pi}{2} - 1 = E(0) - E(1)$, where $E(k)$ is a complete elliptic integral of the second kind (note $c_{10} \simeq \frac{1}{\sqrt{3}} \simeq \gamma \simeq \frac{1}{3}(e-1) \cong \frac{2}{3}(4-\pi) \cong \frac{\ln\pi}{2} \cong \frac{9}{5\pi} \cong \frac{2\pi}{11}$). In (39), we may also express $\mathcal{O}_e(\alpha) = \frac{\alpha^5}{2} + \frac{3}{\pi}\frac{\alpha^4}{\beta} = 1.18213 \times 10^{-11}$, which is within the range of *Lamb shift* $\propto \alpha^5 = 2.069 \times 10^{-11}$ and *Hyperfine splitting* $\propto \alpha^4/\beta = 1.544 \times 10^{-12}$, and both are very sensitive to the magnetic field.

$$\begin{aligned}\frac{|g_e|}{2} &= 1+\frac{\alpha^1}{2\pi}-\frac{\alpha^2}{3\pi^2}+\frac{\alpha^3}{3\pi}-\frac{\alpha^4}{\pi^2}+\frac{\pi-2}{2}\alpha^5 \\ &= \ln\left(e^1\cdot\frac{\exp[\frac{\alpha^3}{2\pi}(\alpha^{-2}+\frac{2}{3})]}{\exp[\frac{\alpha^2}{\pi^2}(\alpha^{+2}+\frac{1}{3})]}\frac{\exp[E(0)\alpha^5]}{\exp[E(1)\alpha^5]}\right)\end{aligned} \tag{42}$$

In (42), the physical definition of α^n is governed by the Rydberg constant ($1\,[\mathrm{Ry}] = hcR_\infty = \frac{1}{2}m_ec^2\alpha^2 = 13.60569253(30)\,[\mathrm{eV}]$)

$$\begin{aligned}4\pi R_\infty &\equiv \frac{\alpha}{a_0}\equiv\frac{\alpha^2}{\lambda\!\!\!^{-}_c}\equiv\frac{\alpha^3}{r_e} \\ &\equiv (\frac{\alpha^4}{a_0r_e})^{\frac{1}{2}}\equiv(\frac{\alpha^5}{\lambda\!\!\!^{-}_cr_e})^{\frac{1}{2}} \\ &\equiv (\frac{\alpha^6}{a_0\lambda\!\!\!^{-}_cr_e})^{\frac{1}{3}}\end{aligned} \tag{43}$$

where $a_0 = \mathrm{e}^2/m_ec^2\alpha^2$ is the Bohr radius, $\lambda\!\!\!^{-}_c = \lambda_c/2\pi = \hbar/m_ec = \hbar c/m_ec^2 = \mathrm{q}^2/m_ec^2 = \mathrm{e}^2/m_ec^2\alpha$ is the reduced Compton wavelength or the classical Planck charge radius, and $r_e = \mathrm{e}^2/m_ec^2$ is a classical electron radius. ($1\,[\mathrm{Hartee}] = 2hcR_\infty = m_ec^2\alpha^2 = 27.21138506\,[\mathrm{eV}]$). From $\alpha^2 \equiv r_e/a_0$, $\alpha \equiv r_e/\lambda\!\!\!^{-}_c \equiv \lambda\!\!\!^{-}_c/a_0$ and $\mu_s \equiv \mu_B g_e/2$, Bohr magneton $\mu_B \equiv \frac{\hbar}{2}\cdot\frac{\mathrm{e}}{m_e}$ is working with α on the reciprocity in **Fig. 6**, where the measurable reduced Compton wavelength serves as the unit.

$$\begin{aligned}\frac{2\mu_B}{c\cdot\mathrm{e}}\frac{1}{a_0} &= \alpha^{+1} &&= \left(\frac{\log e^{\pm1}}{2K\Phi}\frac{1\cdot3\cdot3\cdot5\cdot5\cdot7\cdots}{2\cdot2\cdot4\cdot4\cdot6\cdot6\cdots}\right)^2 \\ \frac{2\mu_B}{c\cdot\mathrm{e}}\frac{1}{\lambda\!\!\!^{-}_c} &= \alpha^0 &&= 1 \\ \frac{2\mu_B}{c\cdot\mathrm{e}}\frac{1}{r_e} &= \alpha^{-1} &&= \left(\frac{2K\Phi}{\log e^{\pm1}}\frac{2\cdot2\cdot4\cdot4\cdot6\cdot6\cdots}{1\cdot3\cdot3\cdot5\cdot5\cdot7\cdots}\right)^2\end{aligned} \tag{44}$$

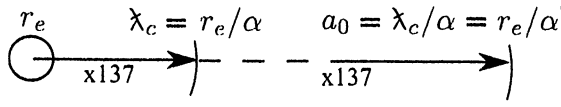

Fig. 6: Bohr magneton $\mu_B \equiv (\hbar/2)(\mathrm{e}/m_e)$ is working with α on the reciprocity

From (43), we can get the simple physical expression

$$\frac{|g_e|}{2} = \ln\left(e^{1}\cdot\frac{\exp[2r_eR_\infty(\frac{a_0}{r_e}+\frac{2}{3})]}{\exp[\frac{4}{\pi}\lambda\!\!\!^{-}_cR_\infty(\frac{r_e}{a_0}+\frac{1}{3})]}e^{\mathcal{O}_e}\right) \tag{45}$$

Using Schwinger and Sommerfeld's $R = \frac{\delta\mu}{\mu} = \frac{\alpha}{2\pi} = \frac{r_e}{\lambda\!\!\!^{-e}_c} = \frac{\mathrm{e}^2}{hc} = \frac{E_e}{E_\phi}$, i.e., the energy ratio between electron E_e and photon E_ϕ

$$\begin{aligned}\frac{|g_e|}{2} &= \ln\left(e^{1}\cdot\frac{\exp\{(2\pi)^2R^3[(2\pi)^{-2}R^{-2}+\frac{2}{3}]\}}{\exp\{\,4\cdot R^2\cdot[(2\pi)^{+2}R^{+2}+\frac{1}{3}]\}}e^{\mathcal{O}_e}\right) \\ &= \underline{1.00115965218072}\end{aligned} \tag{46}$$

which only involves 2π, 4, $\frac{1}{3}$ and $\frac{2}{3}$.

We also get the even more simple muon anomaly formula as

$$\begin{aligned}a_\mu &= \frac{\alpha^1}{2\pi}+\frac{\alpha^2}{\pi^2}-\frac{3\pi\alpha^3}{\pi+1} = \ln\left(\frac{\exp[\frac{\alpha}{2\pi}(\frac{2\alpha}{\pi}+1)]}{\exp[\frac{3\pi\alpha^3}{(\pi+1)})]}\cdot e^{\mathcal{O}_\mu}\right) \\ &= \underline{0.00116592091}183\end{aligned} \tag{47}$$

to match with $a_\mu^{2010} = 0.00116592091(63)$ and $\mathcal{O}_\mu < 10^{-12}$. The muon g/2-factor is

$$\begin{aligned}\frac{|g_\mu|}{2} &= 1+\frac{\alpha^1}{2\pi}+\frac{\alpha^2}{\pi^2}-\frac{3\pi\alpha^3}{\pi+1} \\ &= \ln\left(\frac{\exp\left[\frac{\alpha}{2\pi}(\frac{2\alpha}{\pi}+1)\right]}{\exp\left[\frac{3\pi\alpha^3}{\pi+1}\right]}e^{1+\mathcal{O}_\mu}\right) \\ &= \ln\left(\frac{\exp\left[R(4R+1)\right]}{\exp\left[\frac{(2\pi)^4}{(2\pi+2)}3R^3\right]}e^{1+\mathcal{O}_\mu}\right) = \underline{1.00116592091}183\end{aligned} \tag{48}$$

Looking at the simple and beautiful (41), (42), (45), (46), (47) and (48), Heisenberg is right, without the correct α value, everything is confused! His dream can only be achieved after "fixing the value of $\mathbf{e}^2/hc$". [19] This extremely high precision data matching in (39)-(48) is a strong justification of the theoretical α value yielded by (1).

9 Electroweak and Chirality $e^{\pm i\pi}\alpha^{1/2}$

The β^- decay ($\mathbf{n}^0 \to \mathbf{p}^+ + \mathbf{e}_L^- + \bar{\nu}_e^R$) involves the weak interaction of short range (10^{-17}–10^{-16} m). [48,49] At distances around 10^{-18} meters, the weak force has a similar magnitude to the electromagnetic force; but the weak force is 10,000 times weaker than the electromagnetic force at distances of around 3×10^{-17} m. The electroweak interactions have been merged by $SU(2)_L \otimes U(1)_Y$ with the coupling of charge $\mathbf{Q}$, isospin $\mathbf{I_W}$ and hyper-charge $\mathbf{Y_W}$, linked to the Fermi coupling constant G_F and the Higgs vacuum expectation value v as $G_F \cong \sqrt{2}\mathbf{g}^2/8m_{\mathrm{w}}^2 = 1/(\sqrt{2}v^2)$, by defining $\boldsymbol{e} = \mathbf{g}\sin\theta_w = \mathbf{g}'\cos\theta_w$, where three bosons $\mathrm{W}^{\pm}, \mathrm{W}^0 \sim \mathbf{g}$ in $SU(2)_L$ with the charge coupling $\boldsymbol{e} = \mathbf{g}\sin\theta_w$, and one boson $\mathrm{B}^0 \sim \mathbf{g}'$ in $U(1)_Y$ with $\boldsymbol{e} = \mathbf{g}'\cos\theta_w$.[4] The anomalous electric charge will couple with the weak force in the nucleus as the *electroweak coupling* (per Pythagorean theorem in **Fig. 7**).

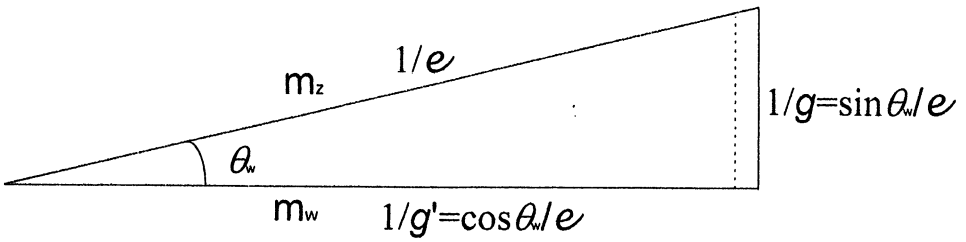

Fig. 7: The charge and mass relationship in the electroweak interactions $SU(2)_L \otimes U(1)_Y$

[4] $G_F = 1.166364(5) \times 10^{-5}[\mathrm{GeV}^{-2}]$, $\mathbf{g}$, $\mathbf{g}'$, $\alpha_{em} = \frac{e^2}{4\pi} = \frac{1}{137} \sim \frac{1}{128}$ and $e = \sqrt{4\pi\alpha} = 0.302822119 \sim 0.312169803$ are in the natural unit. The one-loop *beta*-function in QED is $\beta(e) = \frac{e^3}{12\pi^2}$ or $\beta(\alpha) = \frac{2\alpha^2}{3\pi}$, then $c_4\alpha^2 = \frac{\alpha^2}{3\pi^2} = \frac{\beta_1(\alpha)}{2\pi}$, it is *proportional* to the energy scale.

It is an angle by which spontaneous symmetry breaking rotates the original $W^0 \sim A^3_\mu$ and $B^0 \sim B_\mu$ Goldstone boson plane, producing as a result the vector boson $Z^0 = \cos\theta_w W^0 - \sin\theta_w B^0$ and the massless photon $\gamma = \cos\theta_w B^0 + \sin\theta_w W^0$. [50]

$$\begin{aligned} \begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} &= \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} B^0 \\ W^0 \end{pmatrix} \\ &= \frac{1}{\sqrt{g^2 + g'^2}} \begin{pmatrix} g & g' \\ -g' & g \end{pmatrix} \begin{pmatrix} B_\mu \\ A^3_\mu \end{pmatrix} \end{aligned} \tag{49}$$

After the symmetry breaking, three of the four degrees of freedom in the Higgs field mixing with the three vector bosons (W^+, W^- and Z^0) are only observable as spin components of these weak bosons, which are now massive, while the one remaining degree of freedom becomes the Higgs boson H^0. The Weinberg angle $\cos\theta_w = m_{w^\pm}/m_{z^0}$ or $\tan\theta_w = g'/g$ has to be measured experimentally. [48,49]

The heaviest particle pairs yet produced by electron-positron annihilation in particle accelerators are $W^+ - W^-$ pairs. The heaviest single particle is the Z boson, where $\alpha^{-1}(m_z^2) \cong 128.952(49)$. [50,51] Similar to the format of (1), we get

$$\begin{aligned} \alpha^{1/2}(m_z^2) &= \frac{\pm\log F}{\Phi\pi}(1 - \frac{2}{9}\alpha) \\ &= \frac{\pm\log F}{\Phi\pi}[1 - \frac{2}{9}(\frac{\pm\log e}{\Phi\pi})^2] \\ &= \frac{\log F^{\pm 1}}{\Phi\pi}(1 - \frac{1}{9}\frac{R_K}{Z_0}) \\ &= \pm\frac{\log F}{\Phi\pi}(1 - \alpha\cdot\sin^2\theta_w) \\ &= \frac{\log F^{\pm 1}}{\Phi\pi}(1 + Q_d Q_u \alpha) \\ &= \pm\frac{\log F}{\Phi\pi}\{1 - (\frac{\pm\log e}{\Phi\pi})^2[1 - (\frac{m_w}{m_z})^2]\} \\ &= \alpha_{z'}^{1/2}(1 - \alpha\sin^2\theta_w) = \pm 0.0880614 \end{aligned} \tag{50}$$

and $\alpha^{-1}(m_z^2) = 128.9519596$, where Fransén-Robinson constant[5]

$$\begin{aligned} F &= \int_0^\infty \frac{dt}{\Gamma(t)} \\ &= \lim_{\sigma\to 0}\sigma\sum_{n=0}^{\infty}\frac{1}{\Gamma(\sigma n)} \\ &= \lim_{\sigma\to 0} kE_{\sigma,0}(1) \\ &= 2.807772420 \end{aligned} \tag{51}$$

as the continued gamma function integral for the time interval $\triangle t \to 0$, vs. $e = \sum_{n=0}^{\infty}\frac{1}{\Gamma(n)} = 2.718281828$ as the infinite sum of the discrete gamma function. [2] (50) further approves the correctness of the fine structure constant formula based on $\alpha^{1/2} \approx \pm\frac{\log e}{\Phi\pi}$. From **Fig. 7**, one can get

$$\begin{aligned} e &= \mathrm{g}\sin\theta_w \\ e &= \mathrm{g}'\cos\theta_w \\ e &= \mathrm{g}\mathrm{g}'/(\mathrm{g}^2+\mathrm{g}'^2)^{1/2} \end{aligned} \tag{52}$$

or

$$\mathrm{g}' = \mathrm{g}\tan\theta_w \tag{53}$$

For $\sin^2\theta_w = \frac{2}{9}$ and $e = 0.302822119 \sim 0.312169803$, (52) or (53) yield $\mathrm{g} = 0.642382722 \sim 0.662212155$ and $\mathrm{g}' = 0.343368008 \sim 0.353967286$.

In the $a_e(\alpha)$ equation (39), $c_2 = 1/2\pi$ is originally given by Schwinger, [22] $c_4 = 1/3\pi^2$ is theoretically derived by Kawahara, [47] then $c_6 = 1/3\pi$ and $c_8 = 1/\pi^2$ by following the similar pattern. The *electroweak coupling* may be directly linked to α by $c_2 \sim c_8$ (or $c_2\alpha^1 \sim c_8\alpha^4$) in the electron anomaly formula (43)

---

[5] The Mittag-Leffler function $E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+\beta)} = \frac{1}{2\pi i}\oint_C \frac{t^{\alpha-\beta}e^t}{t^\alpha - z}dt$ is the contour starts and ends at $-\infty$ and circles around the singularities of the integrand, for the Continuous Time Random Walk-CTRW. [48]

$$\eta = \frac{\alpha}{\sin^2\theta_w} = \frac{\mathbf{g}^2}{4\pi} = \frac{c_2 \cdot c_8}{c_6 \cdot c_4} \cdot \alpha = \frac{\mathbf{9}}{\mathbf{2}}\alpha = \frac{\alpha}{0.2222\overline{2}} = \frac{1}{30.452} \tag{54}$$
$$\eta' = \frac{\alpha}{\cos^2\theta_w} = \frac{\mathbf{g}'^2}{4\pi} = \frac{c_2 \cdot c_8 \cdot \alpha}{c_2 \cdot c_8 - c_6 \cdot c_4} = \frac{\alpha}{0.7777\overline{7}} = \frac{1}{106.583}$$

or $\eta_Z = \alpha_Z/\sin^2\theta_w = 28.656$ and $\eta'_Z = \alpha_Z/\cos^2\theta_w = 100.296$.[6] So,

$$G_F \cong \eta\pi/\sqrt{2}m_W^2 = 9\pi\alpha/\sqrt{8}m_W^2 \tag{55}$$
$$= \sqrt{2}\left[\frac{(2n)!!}{(2n+1)!!}\frac{\sin\theta_w}{m_W}\frac{\log e}{K\Phi\pi}\right]^2$$

It links to the Higgs vacuum expectation value

$$v = (\sqrt{2}G_F)^{-1/2} \cong \frac{\sin\theta_w m_W}{\sqrt{\pi\alpha}} \tag{56}$$
$$= \sin\theta_w m_W \sqrt{\pi} K\Phi \ln 10 = 246.2212\,[\mathrm{GeV}]$$

which gives the theoretical value $m_W = \frac{v}{2}\mathbf{g} \simeq 79.0837\,[\mathrm{GeV/c^2}]$ and $m_Z = \frac{v}{2}(\mathbf{g}^2+\mathbf{g}'^2)^{1/2} \simeq 89.6725\,[\mathrm{GeV}]$, compared to the experimental value $m_W = 80.385 \pm 0.015\,[\mathrm{GeV}/c^2]$ and $m_Z = 91.1876 \pm 0.0021\,[\mathrm{GeV}/c^2]$.

(54) yields a reciprocal sum formula $\alpha^{-1} = \eta^{-1} + \eta'^{-1}$ in a similar format to reduced mass, equivalent parallel resistance, or geometrical optics.[7] Compare it to the concave mirror formula $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$ (neglecting aberrations), however, the only possible solution for $\{\alpha, \eta, \eta'\}$ is $R = \infty$, i.e., a flat mirror. There is no space-time curvature or general relativistic correlation for the electroweak theory. It is obvious, since the particle mass is so small, the general relativistic effect does not exist. From (1),

[6] The GUT expressions are $\alpha_1 = \frac{5}{3}\frac{\alpha}{\cos^2\theta_w} \sim \frac{1}{63.95}$; $\alpha_2 = \frac{\alpha}{\sin^2\theta_w} \sim \frac{1}{30.45}$; $\alpha_3 = \alpha_s \sim \frac{1}{8.54}$ and running.

[7] They all involve $\frac{1}{X_{eq}} = \sum_i^n \frac{1}{X_i}$, e.g., Gauss thin lens/concave mirror formula $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$

$$\begin{aligned} \eta^{-1} \quad + \quad \eta'^{-1} \quad &= \quad \alpha^{-1} \\ \left(\sqrt{2}\cdot\frac{K\Phi\pi}{\log e^3}\right)^2 + \left(\sqrt{7}\cdot\frac{K\Phi\pi}{\log e^3}\right)^2 \quad &= \quad \left(\frac{K\Phi\pi}{\log e}\right)^2 \\ \left(\frac{K\pi\sin\theta_w}{\log e^\phi}\right)^2 + \left(\frac{K\pi\cos\theta_w}{\log e^\phi}\right)^2 \quad &= \quad \left(\frac{K\Phi\pi}{\log e}\right)^2 \end{aligned} \tag{57}$$

i.e., $30.4524444 + 106.5835557 = 137.0360003$ or $\frac{1}{0.032838} + \frac{1}{0.0093823} = \frac{1}{0.007297352}$. It obeys Pythagoras Theorem $A^2 + B^2 = C^2$ or the trigonometric identity $\sin^2\theta_w + \cos^2\theta_w = 1$, and fits with Glashow-Weinberg-Salam theory. In this way, the weak-mixing angle is also linked to the $a_e(\alpha)$ equation (39)

$$\begin{aligned} \sin^2\theta_w \quad &= \quad \frac{\alpha}{\eta} = \frac{c_6\alpha^3}{c_2\alpha^1}\frac{c_4\alpha^2}{c_8\alpha^4} = \frac{c_6}{c_2}\frac{c_4}{c_8} = \frac{2\pi}{3\pi}\frac{\pi^2}{3\pi^2} \\ &= \quad 1 - (\frac{m_{\rm w}}{m_{\rm z}})^2 = \frac{2}{3}\times\frac{1}{3} = \frac{2}{9} = 0.2222\overline{2} \end{aligned} \tag{58}$$

where 2/9 is the product of the electric charges of up-quarks 2/3 times down-quarks 1/3. The values of $\frac{1}{3}$ and $\frac{2}{3}$ appeared in the electron anomaly in (46). In the Feynman diagram of the β^- decay, $\mathrm{W}^- \rightarrow \mathrm{e}_L^- + \bar{\nu}_{eR}$, the $d_{-\frac{1}{3}}$ quark changes into a $u_{\frac{2}{3}}$ quark, and also changes flavor. (58) works with CODATA-2010 $\sin^2\theta_w = 0.2223\,(21)$ with $u_r = 9.5 \times 10^{-3}$, and requires further experimental confirmation.

In the β^- decay $\mathbf{n}^0 \rightarrow \mathbf{p}^+ + \mathbf{e}_L^- + \bar{\nu}_{eR}$, the neutron emits a proton, and a *left*-handed electron associated with a *right*-handed electron antineutrino. Chirality projection operators are $P_\pm = \frac{1}{2}(1 \pm \gamma^5)$ where the gamma matrices $\gamma^5 := i\gamma^0\gamma^1\gamma^2\gamma^3$ in *chiral* basis, satisfy $P_+ + P_- = 1$; $P_+^2 = P_+$; $P_-^2 = P_-$; $P_+P_- = P_-P_+ = 0$. Where does this hand $\psi_L^R = \frac{1}{2}(1 \pm \gamma^5)\psi = P_\pm\psi$ originally come from? According to Salam, the physical picture is that "*a photon right circular polarised is 2 neutrinos traveling together and a photon left circular polarised is 2 antineutrinos traveling together.*" [52] In particle physics, the helicity is the projection of the spin

$\vec{S}$ onto the direction of momentum, $\hat{p} = \vec{p}/|\vec{p}|$, i.e.,

$$H = \vec{J} \cdot \hat{p} = \vec{L} \cdot \hat{p} + \vec{S} \cdot \hat{p} = \vec{S} \cdot \hat{p} \tag{59}$$

The invariant transform as $e^{iH\theta}$ under a SE(2) rotation by θ is the helicity H representation. The helicity is equivalent to the chirality operator multiplied by $\frac{1}{2}\hbar$.

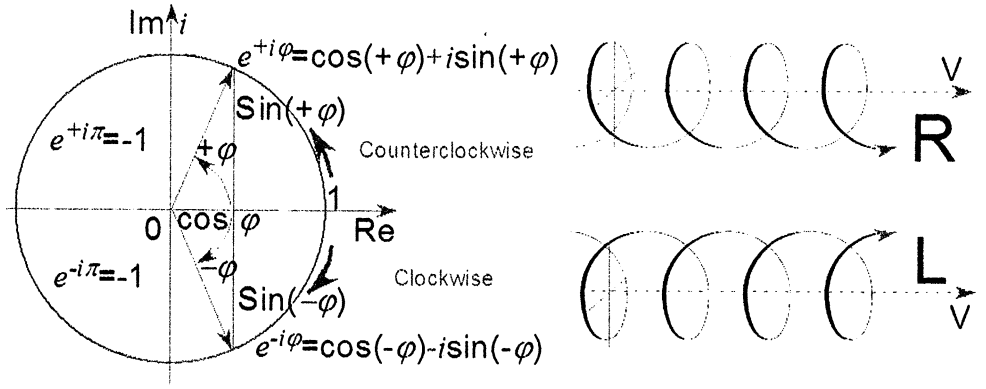

Fig. 8: Reformed Euler identity in 2D, and the right- and left-handed helix in 3D.

In the Euler identity $e^{\pm i\pi} = \phi - \Phi = -1$, regardless of $i\pi$ having $+$ or $-$ sign, $e^{\pm i\pi} \equiv -1$. In the unit-circle (**Fig. 8**), however, two-types of helicities $H = \pm 1$ represent the different rotational directions ("$-$" clockwise and "$+$" counter-clockwise). The "$-$" clockwise spin becomes a 3D left-handed helix rotating around the electron traveling vector (out of paper), and the negatively charged electron has the alternate series for $1/e$. Correspondingly, the positron is right-handed. **Fig. 9**[8] shows that a positron emerging from an electron-positron pair seems to change its charge back to the electron in a very short time. [68]

[8] https://teachers.web.cern.ch/teachers/archiv/HST2002/Bubblech/weigh.html

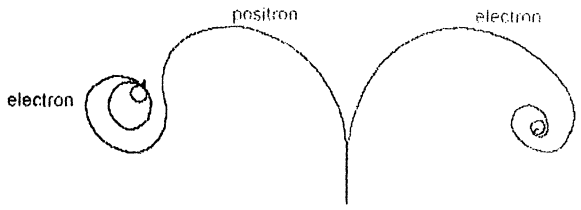

Fig. 9: In the Wilson cloud chamber, a gamma ray photon generates the electron-positron pair spiral in different directions. The positron curves round through about 180 degrees, then seems to change charge sign, and begins to curve in the opposite direction as an electron.

From $e^{\pm i\pi} \equiv -1$, since $\bar{\nu}_{eR}$ takes $e^{+i\pi}$, the electron in weak interaction has the format of $-\alpha^{1/2} = e^{-i\pi}\alpha^{1/2}$, due to Landau *CP* conservation. [53] From the fine structure constant formula (1), multiply $e^{-i\pi}$

$$\begin{aligned} |\alpha^{1/2}| &\equiv e^{-i\pi}\left(\mp\frac{M\phi}{K\pi}\right) \equiv e^{-i\pi}\log e^{\frac{\mp 1}{K\Phi\pi}} \\ &= \log e^{\frac{\pm 1}{K\Phi\pi}} = \log e^{\eta/K\Phi\pi} \end{aligned} \tag{60}$$

where $\eta = \pm 1$ also represents the intrinsic parity of the particle. Parity violation theory in the weak interactions was confirmed by ^{60}Co experiments. [54,55] However, the cause of the parity violation is still not theoretically grounded. It is noticed that (60) temporally changes $e^{\pm 1/K\Phi\pi}$ to $e^{\mp 1/K\Phi\pi}$, i.e., the reverse statistic. Since this could only happen with an electron antineutrino of *right*-handed helicity, the parity violation may only be found in the weak interaction. It also explains why there is no *CP* violation for the strong interaction in QCD.

Not only is the electron *left*-handed, in fact, all fermions, leptons and quarks found in Nature are *left*-handed, and their anti-particles are *right*-handed.

$$\begin{array}{ll} \text{quarks} & \begin{pmatrix} \frac{+2}{3}e \\ \frac{-1}{3}e \end{pmatrix} \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \\ \text{leptons} & \begin{pmatrix} -1e \\ 0\,e \end{pmatrix} \quad \begin{pmatrix} e^- \\ \nu_e \end{pmatrix}_L \quad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}_L \quad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}_L \end{array} \tag{61}$$

This must be due to the spacetime properties of the electromagnetic wave. **The electromagnetic field obeys the *right*-handed rule, correspondingly, the particle properties must obey the *left*-handed rule.** The fundamental natural philosophy is based on the principle of symmetry/asymmetry. The fine structure constant formula is derived based on the symmetry~asymmetry principle, which is why and how the universe is so beautiful.

10 Topology of Quark Charge and Confinement

In **Fig 2.1** of the first paper ((I) The fine structure constant), the electromagnetic field with $\mathbf{E} : \mathbf{H} = \mathbf{B} : \mathbf{D} = 376.730 : 1$ is a near-linear polarization (like a flat belt). [2] The free space impedance is an electric-to-magnetic ratio $Z_0 = |\mathbf{E}|/|\mathbf{H}| = \mu_0 c = (\mu_0/\varepsilon_0)^{1/2} = 376.730\,\Omega$, while the particle impedance is $R_K = h/\mathbf{e}^2 = g/\mathbf{e} = Z_0/2\alpha = 205812.807\,\Omega$, and their ratio is $R_K/Z_0 = 1/2\alpha = 68.518$. **Compared to the free space, the particle's electric property becomes stronger or the magnetic property becomes weaker; in other words, the particle has been "charged". The fine structure constant is the coupling between "charged" and "uncharged" zones.** Obviously, the space-time is twisted or distorted by the existence of the particle. Charged particles with high quantum impedance R_K are surrounded by a low impedance of free-space Z_0, and the coupling constant between two

zones (Fermionic/Bosonic) is the fine structure constant α. In this way, a charged particle becomes the spacetime photon-trap of electromagnetic wave, i.e., "tie knot."

There are six types of quarks: up, down, strange, charm, bottom, and top, known as flavors. In the strong interaction, there are two mysteries on the quark charge - triplets and confinement. [15, 56–58] They may involve geometric topology in the form of a vector bundle. A non-trivial *fiber bundle* (e.g., the Möbius strip) is a line bundle over the 1-sphere S^1. The electromagnetic wave in 4D space-time can be polarized and twisted to form a Möbius strip in the particle's generation, with a reversed process as the particle annihilates (**Fig. 10**). This is a physical illustration for the conversion between the photon and particle.

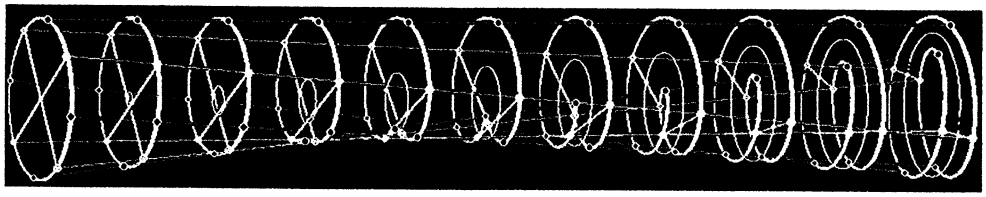

Fig. 10: The electromagnetic wave can be polarized into a Möbius strip of particle. Notice the spin-1 ($h/2\pi$) changing to spin-$\frac{1}{2}$ ($h/4\pi$), and the cardioid section in the middle.

The topological illustration of the electric charge is linked to the Möbius strip, either as a twisted cylinder or a lemniscate belt, depending on the length of the strip. There are two types of Möbius strips depending on the direction of the half-twist: clockwise and counterclockwise. It is therefore chiral, i.e., "handed". The Möbius strip is related to the fiber-bundle, and the "Zitterbewegung" per Dirac. The term "gauge" is more often used in particle physics. It can be seen as a torus knot from a different angle (**Fig. 11**). Since its surface is not orientable, a round trip must take two cycles (4π) to return to the starting point. This verifies the particles with spin-1/2 ($\hbar/2 = h/4\pi$).

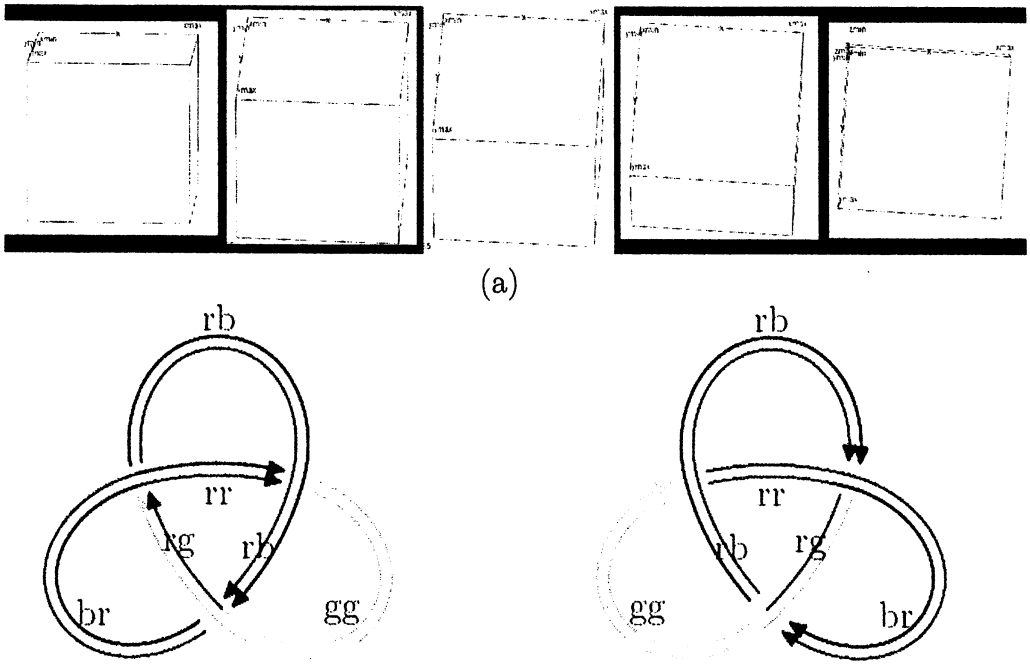

Fig. 11: The Möbius strip can be linked to (a) a torus knot or trifoil knot in different angles from front-view to top-view (b) tricolorability (r-g-b) and chiral (left- or right-handed).

In the cylindrical polar coordinates (r, θ, h), an unbounded Möbius strip is

$$\log(r)\tan(\frac{\theta}{2}) = h \tag{62}$$

Let $h = \alpha^{1/2} = \log(e^{\pm 1/K\Phi\pi})$ in (1) and at $\theta = \pm\pi/2$, one naturally gets

$$\log(r) = \log(e^{\pm 1/K\Phi\pi}) = |\alpha^{1/2}| \tag{63}$$

i.e. $r_{\pm\pi/2} = e^{\pm 1/K\Phi\pi}$.

In $\mathbf{R}^3 - t$ (4D), a Möbius strip can be presented as a group of parametric equations with $\alpha^{1/2}$ (or $\alpha_z^{1/2}$ at high energy scale)

$$
\begin{aligned}
x(u,v,t) &= (\alpha^{1/2} \pm \frac{u}{2}\cos(\frac{v}{2}))\cos(v+\omega_0 t \pm 2n\pi) \qquad (64)\\
y(u,v,t) &= (\alpha^{1/2} \pm \frac{u}{2}\cos(\frac{v}{2}))\sin(v+\omega_0 t \pm 2n\pi)\\
z(u,v,t) &= \frac{u}{2}\cdot\sin(\frac{v}{2}+\omega_0 t \pm 2n\pi)
\end{aligned}
$$

where $-\alpha^{1/2} \leq u \leq +\alpha^{1/2}$, $0 \leq v \leq 2\pi$, $\omega_0 = constant$, and $0 < t < \infty$; "+" for *right*-handed or "–" for *left*-handed. This creates two types of live-rotating Möbius strips of width $\alpha^{1/2}$, whose immobile central circle lies on the $x-y$ plane and is centered at (0,0,0) with radius $\alpha^{1/2}$. The angle parameter u runs around the strip while v moves from one edge to the other. (64) is a *torus* in 4D space-time when $\omega_0 t$ is turning. However, charge is timeless as $\sin(\theta) = \sin(2n\pi \pm \theta)$,[9] so it can be plotted in 3D as **Fig. 12**. In this way, a particle with linear speed αc, charge $e = (\alpha\hbar c)^{1/2}$ and spin $\hbar/2$ can be illustrated as a *left*-handed Möbius strip. Obviously, its mirror image is *right*-handed.

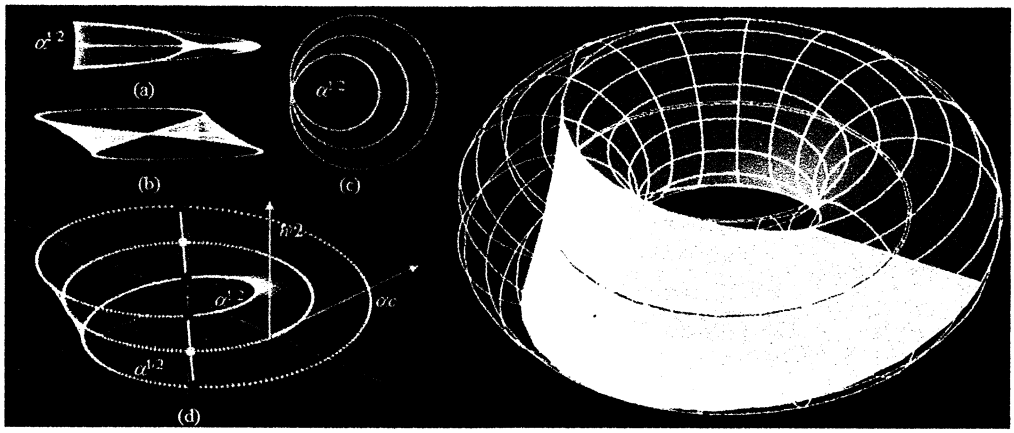

Fig. 12: The $\alpha^{1/2}$ illustration of equation (64) in 3D: (a) XOZ, (b) YOZ, (c) XOY, (d) Isometric view shows R-G-B flipped on the edge, and 4D torus.

[9]Mach's principle that a rotation leaves no trace at all, which also works for the creation of mass.

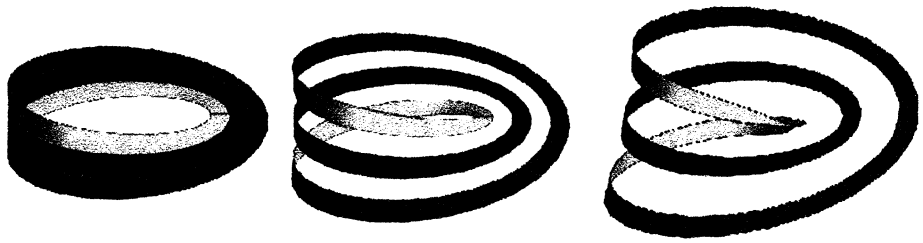

Fig. 13: Graph of (64). Cutting 1/3 of a Möbius strip into triplet charge: one immobile middle ring (black) and another confinement ring.

Cutting this Möbius strip at 1/3 width of $\alpha^{1/2}$ will produce two conjoined rings in **Fig. 13**: one immobile ring in the middle equal to -1/3, and another equal to +2/3 in a limited range motion. The sign alternates in each cycle of the Möbius strip, so, for the two cyclic motion rings $(-1)^2 = +1$. This imitates the quark charge, and the conjoined or paradromic rings also illustrate why there is no free quark (the color confinement, or the asymptotic freedom).

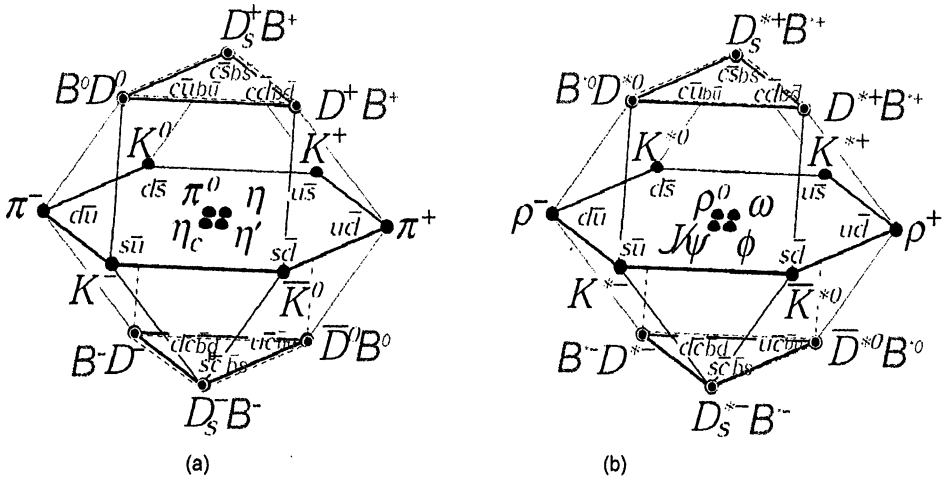

Fig. 14: Quark-antiquark pair of u, d, s, c, b makes Mesons in extended $SU(3)_c^* = (|u\rangle, |d\rangle, |s\rangle) \oplus (|c\rangle, |b\rangle, |t\rangle)$ (**a**) spin-0; (**b**) spin-1.

In group theory, $SU(3)_c = (|u\rangle, |d\rangle, |s\rangle)$ organizes mesons in octets: $\mathbf{3}\otimes\overline{\mathbf{3}} = \mathbf{8}\oplus\mathbf{1}$, and baryons in octets or decuplets: $\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3} = \mathbf{10}_S\oplus\mathbf{8}_M\oplus\mathbf{8}_M\oplus\mathbf{1}_A$. The algebraic sum of two spin-1/2s becomes spin-0 or 1 ($\frac{1}{2}\mp\frac{1}{2}$). This model fits the Mesons (**Fig. 14**(a)-(b)) as a valence quark-antiquark pair (e.g., $\pi^+ = \bar{d} + u \Rightarrow \frac{1}{3} + \frac{2}{3} = +1$; $\pi^- = d + \bar{u} \Rightarrow -\frac{1}{3} - \frac{2}{3} = -1$; $\pi^0 = \left(d\bar{d} \pm u\bar{u}\right)/\sqrt{2} \Rightarrow 0$). These pions are the unstable transform center between Mesons, Baryons and Leptons.

In the weak interactions, heavier Mesons and Baryons decay to pions, then pions decay to Leptons or γ-rays. The decay rate from a vector meson to leptons is proposed by van Royen-Weisskopf [55]

$$\Gamma_\gamma = 16\pi(\alpha_{em}^2/M_V)^2|\sum_i a_i Q_i|^2|\psi(0)|^2 \tag{65}$$

If we assume that $|\psi(0)|^2/M_V^2$ is the same for ρ, ω, ϕ (masses are 770 MeV, 780 MeV, and 1020 MeV respectively), then expect $\Gamma_\gamma(\rho) : \Gamma_\gamma(\omega) : \Gamma_\gamma(\phi) = 9 : 1 : 2$ to match the measured $(8.8\pm2.6) : 1 : (1.7\pm0.4)$. Using (1), (65) can be linked to the *vacuum* conductivity in a squared format

$$\begin{aligned}\Gamma_\gamma &= \frac{G_0}{\sigma_0}(\alpha_{em}\cdot\frac{e}{M_V})^2|\sum_i a_i Q_i|^2|\psi(0)|^2 \\ &= \left[\frac{2}{\pi}\frac{\log e^{\pm1}}{K\Phi}\right]^2\left[\frac{\alpha_{em}e}{M_V}\right]^2|\sum_i a_i Q_i|^2|\psi(0)|^2\end{aligned} \tag{66}$$

where $\alpha_{em} = e^2/4\pi = 1/137$, M_V is the mass of the vector meson, Q is the charge of the quarks, and $\psi(0)$ is the wavefunction for the two quarks to overlap each other. (66) shows that the meson decay involves the *vacuum/quantum* conductivity ratio $G_0/\sigma_0 = 4\alpha = 2Z_0/R_k$ and the charge/mass ratio e/M_V. The physical picture becomes more and more clear, this is how the fine structure constant regulates the space-time properties. Quantum mechanics is not a garbage field. Like Einstein said, "That the Lord should play dice, all right; but that He should gamble according to definite rules, that is beyond me." [67] It is noticed that b-quarks make B-mesons, and t-quarks only decay to W-bosons and b-quarks.

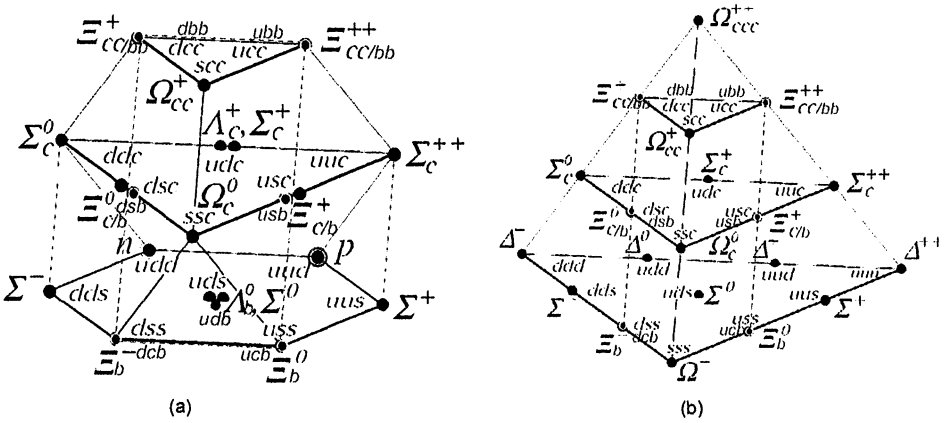

Fig. 15: Combinations of three u, d, s, c, b quarks make baryons in extended $SU(3)_c^*$ (**a**) spin-1/2 octet where the stable proton p and quasi-stable neutron n make atoms; (**b**) spin-3/2 decuplet. Note b quark makes $\Lambda \sim \Xi$ Baryons.

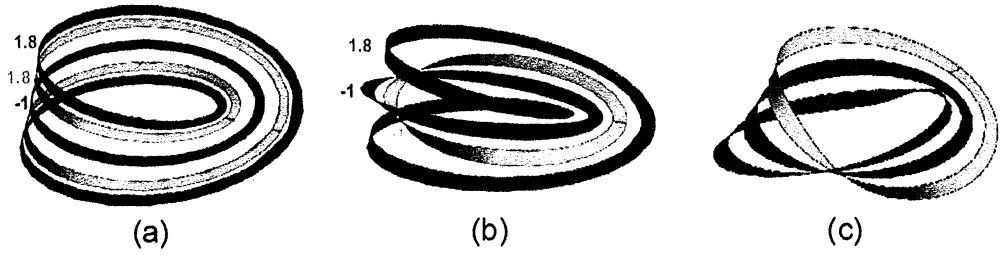

Fig. 16: Cutting the Möbius strip into three paradromic rings (a) $\mathbf{p}^+ - uud$ $2\times(2/3)-1/3=1$, (b) $\mathbf{n}^0 - ddu$ $2\times(-1/3)+2/3=0$, (c) Δ^--ddd $3\times(-1/3)=-1$ or Δ^{++}-uuu $3\times(2/3)=2$.

Baryons in **Fig. 15** (a)-(b) are made by three quarks with spin-1/2 or 3/2, and their electric charge can be $0, \pm 1, \pm 2$ (e.g., proton as $uud \Rightarrow 2\times(2/3)-1/3=1$ and neutron as $ddu \Rightarrow 2\times(-1/3)+2/3=0$). They can also be illustrated by cutting the Möbius strip into three paradromic rings ($2\times(2/3)-1/3$ or $2\times(-1/3)+2/3$) in **Fig. 16** (a)-(c)). Note that

only **Fig. 16** (a) is a regularly cut Möbius strip symbolizing a stable proton, and (b)-(c) are made of composite Möbius strips, symbolizing the unstable particles.

The cutting of the Möbius strip in **Fig. 13** and **Fig. 16** can be simply tested with bare hands. These models can help in understanding the quark model in QCD, however, one must really study QCD to learn how the strong interaction works.

11 The g-factors of Proton and Neutron

The particle's g-factor is given as $g_i = 2\mu_i/\mu_N$. The quark model was proposed after the proton $g_p = 5.585$ and neutron $g_n = -3.826$ were confirmed, indicating that they are composite nucleons made of sub-particles. [71] In the Standard Model, the proton and neutron are reinterpreted as the isospin symmetry of the up and down quarks. The nucleon doublet states are seen to be linear combinations of products of 3-particle isospin doublet states and spin doublet states. The spin-up proton and neutron wave function are described by

$$
\begin{aligned}
|p\uparrow\rangle &\equiv \frac{1}{3\sqrt{2}}\begin{pmatrix} |duu\rangle & |udu\rangle & |uud\rangle \end{pmatrix}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}\begin{pmatrix} |\downarrow\uparrow\uparrow\rangle \\ |\uparrow\downarrow\uparrow\rangle \\ |\uparrow\uparrow\downarrow\rangle \end{pmatrix} \\
|n\uparrow\rangle &\equiv \frac{1}{3\sqrt{2}}\begin{pmatrix} |udd\rangle & |dud\rangle & |ddu\rangle \end{pmatrix}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}\begin{pmatrix} |\downarrow\uparrow\uparrow\rangle \\ |\uparrow\downarrow\uparrow\rangle \\ |\uparrow\uparrow\downarrow\rangle \end{pmatrix}
\end{aligned}
\tag{67}
$$

where $|u\rangle$ and $|d\rangle$ are the u and d quark flavor eigenstates, while $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of S_z. Although these superpositions are the correct way of denoting a proton and neutron in terms of quark flavour and spin eigenstates, however, they are often referred as uud for proton and udd for neutron.

Quark model $SU(6) = (|u\rangle, |d\rangle, |s\rangle) \otimes (|\frac{1}{2}\rangle, |-\frac{1}{2}\rangle)$ only gives a scalar

explanation for the ratio of neutron and proton magnetic moments [59]

$$\frac{\mu_p}{\mu_n} = \frac{g_p}{g_n} = \frac{\langle\psi_p|\mu|\psi_p\rangle}{\langle\psi_n|\mu|\psi_n\rangle} = \frac{\langle P,\frac{1}{2}|Q\sigma_3|P,\frac{1}{2}\rangle}{\langle N,\frac{1}{2}|Q\sigma_3|N,\frac{1}{2}\rangle} = \frac{36/36}{-24/36} = -\frac{3}{2} \qquad (68)$$

In $SU(6)$ $\{\mathbf{6}\otimes\mathbf{6}\otimes\mathbf{6} = \mathbf{56}_S \oplus \mathbf{70}_M \oplus \mathbf{70}_M \oplus \mathbf{20}_A\}$, $\frac{\mu_p}{\mu_n} = \frac{\mathbf{33}}{\mathbf{12}} = -2.75$ is also discussed. [59][10]

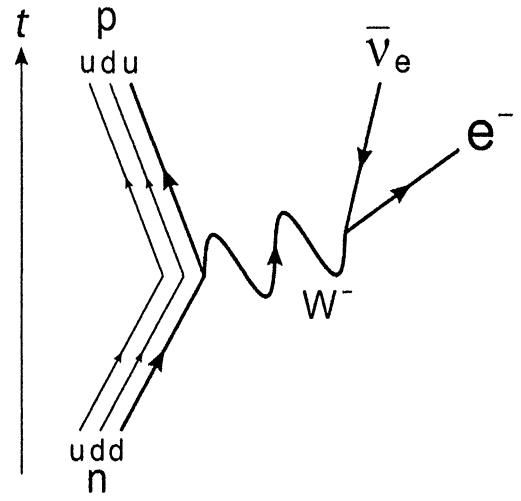

Fig. 17: The d-quark changes to u-quark when the neutron releases the proton and electron in beta-decay $n^0 \to p^+ + W^- \to p^+ + e^- + \overline{\nu}_e$. [65]

The β-decay involves the proton and neutron, $n^0 \to p^+ + W^-$ vertex factor: $(1-\gamma^5) \Rightarrow (c_V - c_A\gamma^5)$ where the measured $c_V = 1.000$ is the

[10] In the textbook of "quantum Mechanics-symmetries", p284-285 [64] From $SU(6)$, $\mu_p = \frac{3}{18}[4(\mu_u+\mu_u-\mu_d)+(\mu_u-\mu_u+\mu_d)+(-\mu_u+\mu_u+\mu_d)] = \frac{3}{18}(8\mu_u-2\mu_d) = \underline{\frac{1}{3}(4\mu_u-1\mu_d)}$ for proton, and by simply interchange u and d. they get neutron $\mu_n = \frac{3}{18}(8\mu_d-2\mu_u) = \underline{\frac{1}{3}(4\mu_d-1\mu_u)}$ (This is obviously wrong). Next, asumming the magnetic moments of quarks are proportional to their charge bur neglect mass differences, $\mu_d = \frac{-1/3}{2/3}\mu_u = -\frac{1}{2}\mu_u$;

So, $\left(\frac{\mu_n}{\mu_p}\right)_{\text{theor}} = \underline{\frac{4\mu_d-1\mu_u}{4\mu_u-1\mu_d}} = -\frac{2}{3} = -0.6\overline{6} = \underline{-\frac{1}{1.5}}$, while $\left(\frac{\mu_n}{\mu_p}\right)_{\text{exp}} = -0.685 = \underline{-\frac{1}{1.459840}}$.

From the experimental values, $\mu_u = \frac{1}{5}(\mu_n+4\mu_p) = 1.852\mu_0$ and $\mu_d = \frac{1}{5}(\mu_p+4\mu_n) = -0.972\mu_0$, where $\mu_0 = \frac{e\hbar}{2Mc}$ and M is the proton mass.

correction to the vector 'weak charge' and $c_A = 1.270 \pm 0.003$ is the correction to the axial vector 'weak charge'. It also changes the d-quark to u-quark as **Fig. 17.** $\frac{1}{4}(c_V^2 + 3c_A^2) = 1.459675$ gives the free neutron lifetime $\tau = 1316s/1.46 = 901$s. [65] However, this is an experimental result. The accurate theoretical calculation is still nowhere to be seen.

In CODATA-2010, many experimental data points to the same number,

$$\begin{aligned}
\frac{\mu_p}{\mu_n} &= \frac{1.410606743(33) \times 10^{-26}[\mathrm{JT}^{-1}]}{-0.96623647(23) \times 10^{-26}[\mathrm{JT}^{-1}]} \\
= \frac{\gamma_p}{\gamma_n} &= \frac{2.675222005(63) \times 10^{8}[\mathrm{s}^{-1}\mathrm{T}^{-1}]}{-1.83247179(43) \times 10^{8}[\mathrm{s}^{-1}\mathrm{T}^{-1}]} \\
= \frac{\gamma_p/2\pi}{\gamma_n/2\pi} &= \frac{42.5774806(10)[\mathrm{MHz}\cdot\mathrm{T}^{-1}]}{-29.1646943(69)[\mathrm{MHz}\cdot\mathrm{T}^{-1}]} = \\
\frac{\mu_p/\mu_B}{\mu_n/\mu_B} = \frac{g_p/2\beta}{g_n/2\beta} &= \frac{1.521032210(12) \times 10^{-3}}{-1.04187563(25) \times 10^{-3}} = \\
\frac{g_p}{g_n} = \frac{2\mu_p/\mu_N}{2\mu_n/\mu_N} &= \frac{\mathbf{5.585694713}(46)}{\mathbf{-3.82608545}(90)} = \\
\frac{g_p/2}{g_n/2} = \frac{\mu_p/\mu_N}{\mu_n/\mu_N} &= \frac{\mathbf{2.792847356}(23)}{\mathbf{-1.91304272}(45)} = \mathbf{-1.45989806}(34)
\end{aligned} \tag{69}$$

Since the g-factor of an electron with a whole Möbius strip is about $|g_e|/2 = 1 + a_e$, assume that the $g/2$ of those immobile rings is also close to ± 1, and the $g/2$ of a cut confinement ring of two cycles is close to 2 (**Fig. 15**(a)-(b)). We already know that particle mass does not affect the g-factors or the anomaly very much. Since $\mu_i = \frac{|g_i|}{2}\frac{e}{m_i}s$, from the toy-model of $\mathbf{p}^+(uud)$ and $\mathbf{n}^0(udd)$ in **Fig. 15**(a)-(b), [60] only considering the quark charge, [61,64]

$$\begin{aligned}
|\mu_p| &= +\frac{2}{3}\mu_u + \frac{2}{3}\mu_u - \frac{1}{3}\mu_d = \frac{1}{3}(4\mu_u - 1\mu_d) \\
|\mu_n| &= +\frac{2}{3}\mu_u - \frac{1}{3}\mu_d - \frac{1}{3}\mu_d = \frac{1}{3}(2\mu_u - 2\mu_d)
\end{aligned} \tag{70}$$

The anomaly and the quark electric charge are related to the g-2 of proton or neutron

$$\begin{aligned}
|\mu_p| &= +\frac{2}{3}\mu_u + \frac{2}{3}\mu_u - \frac{1}{3}\mu_d = \frac{1}{3}(4\mu_u - 1\mu_d) \\
&= \frac{|g_p|}{2} \cdot |\mu_0| = (g_p^I + |a_p|)|\mu_0| = (1 + |a_p|)|\mu_0| \\
&= 2.792847356|\mu_0| \\
&= (1 + \underline{1.792847356})|\mu_0| \\
|\mu_n| &= +\frac{2}{3}\mu_u - \frac{1}{3}\mu_d - \frac{1}{3}\mu_d = \frac{1}{3}(2\mu_u - 2\mu_d) \\
&= \frac{|g_n|}{2} \cdot |\mu_0| = (g_n^I - |a_n|)|\mu_0| = (0 - |a_n|)|\mu_0| \\
&= -1.91304272 \cdot |\mu_0| \\
&= (0 + \underline{1.91304272})|\mu_0|
\end{aligned} \tag{71}$$

where μ_p is the same result previously discussed in $SU(6)$, and μ_n uses the same concept of charge conservation. In (71), the values of $g_p^I = 1$ and $g_n^I = 0$ are according to their electric charge, therefore, the proton and neutron anomalies are $a_p = 1.792847356$ and $a_n = 1.91304272$. Let $|\mu_0| = \frac{e\hbar}{2Mc} \equiv 1$, solve (71) and get the closest fitting results as

$$\begin{aligned}
\mu_u/|\mu_0| &= g_u/2 = +1.836325996 \\
&\cong +2 - \frac{6}{\pi}\alpha^{1/2} - \underline{\frac{11}{\pi}\frac{16}{9e^2}\alpha^{3/2}} \\
&= 1.836325\underline{996074} \\
\mu_d/|\mu_0| &= g_d/2 = -1.033238084 \\
&\cong -1 - \frac{11}{9\pi}\alpha^{1/2} - \underline{\frac{33}{37}\alpha^{5/2}} \\
&= -1.03323\underline{808399}
\end{aligned} \tag{72}$$

In (72), the first and second correlation is proportional to $\alpha^{n+1/2}$. The series of $\sum c_{2n+1}\alpha^{n+1/2}$ in the Heisenberg S-matrix (20) are used here

for QCD, which has been "cut-off" ("sweep the difficulties under the rug") by QED. In this way, the g-factors of Leptons and Baryons use different parts of the divergent perturbation series in the S-matrix, in terms of *odd* and *even* numbers in (20). Leptons involve an *even* number of electrons $(2, 4, 6, ...)$ and α^n, while Baryons involve an *odd* number of charges $(1, 3, 5, ...)$ and $\alpha^{n/2}$. In the strong-electroweak coupling, the baryons anomaly is $a_B(\alpha^{n+1/2}) = \sum_{n=1}^{N} c_{2n+1}\alpha^{n+1/2}$. From (71) and (72), the anomaly of proton and neutron are a good match with the directly measured $|g_p|/2 = \mathbf{2.792847356}(23)$ for proton, and $|g_n|/2 = \mathbf{1.91304272}(45)$ for neutron

$$
\begin{aligned}
|a_p| &= \mathbf{2} + \frac{1}{\pi}\left[\frac{\mathbf{1}}{\mathbf{3}} + \frac{\mathbf{2}}{\mathbf{27}} - \mathbf{2^3}\right]\alpha^{1/2} \underline{- \frac{\mathbf{4}}{\mathbf{3}}\frac{\mathbf{11}}{\pi}\frac{\mathbf{16}}{9e^2}\alpha^{3/2} + \frac{\mathbf{1}}{\mathbf{3}}\cdot\frac{\mathbf{33}}{\mathbf{37}}\alpha^{5/2}} \\
&= \ln\left(\frac{e^2\cdot\exp[\frac{\mathbf{1}}{\mathbf{3}}(\frac{\mathbf{11}}{\mathbf{9}\pi}\alpha^{1/2} + \frac{\mathbf{33}}{\mathbf{37}}\alpha^{5/2})]}{\exp[\frac{\mathbf{1}}{\pi}(8\alpha^{1/2} + \frac{\mathbf{11}}{e^2}(\frac{4}{3})^3\alpha^{3/2})]}\right) \\
&= \mathbf{2} + \left(\frac{\mathbf{11}}{\mathbf{27}} - \mathbf{8}\right)\frac{\alpha^{1/2}}{\pi} \underline{+\mathcal{O}_p} \qquad (73) \\
&= \underline{1.792847355}486 \\
|a_n| &= \mathbf{2} + \frac{1}{\pi}\left[\frac{\mathbf{2}}{\mathbf{3}} + \frac{\mathbf{4}}{\mathbf{27}} - \mathbf{2^2}\right]\alpha^{1/2} \underline{- \frac{\mathbf{2}}{\mathbf{3}}\cdot\frac{\mathbf{11}}{\pi}\frac{\mathbf{16}}{9e^2}\alpha^{3/2} + \frac{\mathbf{2}}{\mathbf{3}}\cdot\frac{\mathbf{33}}{\mathbf{37}}\alpha^{5/2}} \\
&= \ln\left(\frac{e^2\cdot\exp[\frac{\mathbf{2}}{\mathbf{3}}(\frac{\mathbf{11}}{\mathbf{9}\pi}\alpha^{1/2} + \frac{\mathbf{33}}{\mathbf{37}}\alpha^{5/2})]}{\exp[\frac{\mathbf{1}}{\pi}(4\alpha^{1/2} + \frac{\mathbf{11}}{e^2}(\frac{4}{3})^3\cdot\alpha^{3/2})]}\right) \\
&= \mathbf{2} + \left(\frac{\mathbf{22}}{\mathbf{27}} - \mathbf{4}\right)\frac{\alpha^{1/2}}{\pi} \underline{+\mathcal{O}_n} \\
&= \underline{1.91304271}998
\end{aligned}
$$

where $\alpha^{n+1/2}$ involves the *odd number* charge interaction $\mathbf{e}^1, \mathbf{e}^3, \mathbf{e}^5, \cdots$ (e.g., $p^+(uud)$ and $n(udd)$ in three charged quarks, plus W^- and $\mathbf{e}^-$ in the neutron decay as **Fig. 16**). It is noticed that $\frac{\mathbf{1}}{\mathbf{3}}$ and $\frac{\mathbf{2}}{\mathbf{3}}$ appeared in (41), (42), (45) and (46), also $\frac{\mathbf{2}}{\mathbf{9}}$ in (50), and (54) . From (73), the g/2 factors for proton and neutron are

$$\frac{1}{2}\begin{bmatrix} |g_p| \\ |g_n| \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \left\{ \begin{bmatrix} \frac{11}{27} \\ \frac{22}{27} \end{bmatrix} - \begin{bmatrix} 8 \\ 4 \end{bmatrix} \right\} \frac{\alpha^{\frac{1}{2}}}{\pi} - \underline{\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} \frac{22}{9\pi} \frac{16}{e^2} \alpha^{\frac{3}{2}} + \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} \frac{33}{37} \alpha^{\frac{5}{2}}}$$

$$= \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \frac{1}{\pi} \cdot \left\{ \left(1 + \frac{1}{3} \cdot \frac{2}{3} \right) \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} - \begin{bmatrix} 2^3 \\ 2^2 \end{bmatrix} \right\} \cdot \alpha^{\frac{1}{2}} + \begin{bmatrix} \mathcal{O}_p \\ \mathcal{O}_n \end{bmatrix}$$

$$= \ln \left(\exp \begin{bmatrix} 3 \\ 2 \end{bmatrix} \cdot \frac{\exp \left\{ \frac{11}{9\pi} \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} \alpha^{\frac{1}{2}} \right\}}{\exp \left\{ \frac{1}{\pi} \begin{bmatrix} 8 \\ 4 \end{bmatrix} \alpha^{\frac{1}{2}} \right\}} \cdot \underline{\exp \begin{bmatrix} \mathcal{O}_p \\ \mathcal{O}_n \end{bmatrix}} \right) \tag{74}$$

$$= \begin{bmatrix} 2.792847355\underline{48} \\ 1.913042719\underline{98} \end{bmatrix}$$

There is $(3, 8)$ for the stable proton and $(2, 4)$ for the decaying free neutron in (74). It may be related to the gluons represented by the 8 Gell-Mann matrices or something else. (74) can be physically expressed as

$$\frac{1}{2}\begin{bmatrix} |g_p| \\ |g_n| \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \frac{\mathrm{e}}{\pi \mathrm{q_p}} \left\{ \sin^2 \theta_w \begin{bmatrix} \frac{11}{6} \\ \frac{11}{3} \end{bmatrix} - \begin{bmatrix} 2^3 \\ 2^2 \end{bmatrix} \right\} \tag{75}$$

$$\underline{\mp 2 \begin{bmatrix} Q_u \\ Q_d \end{bmatrix} \left(\frac{4}{3e} \right)^2 \frac{11}{\pi \mathrm{q_p}} \frac{\lambda\!\!\!^{-}_c}{a_0} \mp 1 \begin{bmatrix} Q_d \\ Q_u \end{bmatrix} \frac{33}{37} \frac{1}{\mathrm{q_p}} \frac{r_e}{a_0}}$$

$$= \ln \left(\frac{\exp \begin{bmatrix} 3 \\ 2 \end{bmatrix} \cdot \exp \left\{ \frac{\mathrm{e}}{\pi \mathrm{q_p}} \left(\sin^2 \theta_w \begin{bmatrix} \frac{11}{6} \\ \frac{11}{3} \end{bmatrix} - \begin{bmatrix} 2^3 \\ 2^2 \end{bmatrix} \right) \right\}}{\exp \left\{ \underline{\pm 2 \begin{bmatrix} Q_u \\ Q_d \end{bmatrix} \left(\frac{4}{3e} \right)^2 \frac{11}{\pi \mathrm{q_p}} \frac{\lambda\!\!\!^{-}_c}{a_0} \pm 1 \begin{bmatrix} Q_d \\ Q_u \end{bmatrix} \frac{33}{37} \frac{1}{\mathrm{q_p}} \frac{r_e}{a_0}} \right\}} \right)$$

$$= \begin{bmatrix} 2.792847\underline{578} \\ 1.913042\underline{844} \end{bmatrix}$$

where the quark charges are $Q_u = +\frac{2}{3}\mathrm{e}$ and $Q_d = -\frac{1}{3}\mathrm{e}$. (75) is comparable to the *beta*-function in QCD. Since $SU(6) = SU(3) \times SU(2)$ has the total number of generators $8 + 3 = 11$ on 6 dimensional tensor product space, $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$, which involve the number in *beta*-function of the

asymptotic freedom $\beta_1(\alpha_s) = \frac{\alpha_s^2}{\pi}(\frac{-11}{6}N + \frac{1}{3}n_f) = \frac{\alpha_s^2}{3\pi}(\frac{-11}{2}N + n_f)$. [57,58][11]

The toy-model of asymptotic freedom in **Fig. 13**(a)-(b) requires g_p and g_n in different signs, i.e., $g_p/2 = \mathbf{2.7928}$ and $g_n/2 = \mathbf{-1.9130}$ for particle ($g_{\overline{p}}/2 = \mathbf{-2.7928}$ and $g_{\overline{n}}/2 = \mathbf{1.9130}$ for anti-particle). In fact, all lepton g_i values are *negative* (g_e = -2.00231930436153(53); g_μ = -2.0023318418(13) and $g_\tau = -2.008$), which is ignored due to the anomaly always being a positive number. The *negative* sign means that the particle has a tendency to align anti-parallel to a magnetic field. The neutron $g_n/2 = -1.91304272(45)$ is also negative, and only the proton $g_p/2 = 2.792847356(23)$ is a *strange* positive. This gives the ratio in (69)

$$\begin{aligned} \frac{\mu_p}{\mu_n} &= \frac{g_p/2}{g_n/2} = \frac{g_p}{g_n} = \frac{4\mu_u - 1\mu_d}{2\mu_d - 2\mu_u} \\ &= \mathbf{-1.459898082} \cong \zeta(\frac{1}{2}) \end{aligned} \tag{76}$$

which is close to the Riemann zeta function $\zeta(\frac{1}{2}) = \zeta(s) = -1.46035450880$. The g-factors of proton and neutron g_p and g_n have different signs. Neutrons with negative g_n will decay after leaving the nuclear quark confinement.

The anomaly of Leptons and Baryons in (41), (47) and (73) have a similar logarithmic format, and can be uniformly expressed as the information entropy

$$\begin{pmatrix} a_e & a_p \\ a_\mu & a_n \end{pmatrix} = \begin{pmatrix} \mathbf{0.0011596521807} & \mathbf{1.792847578} \\ \mathbf{0.0011659207972} & \mathbf{1.913042844} \end{pmatrix} = \tag{77}$$

$$\begin{pmatrix} \ln\left(e^{\mathbf{0}} \cdot \frac{\exp[\mathbf{A}(\alpha^{-2} + \frac{\mathbf{2}}{\mathbf{3}})]}{\exp[\mathbf{B}(\alpha^{+2} + \frac{\mathbf{1}}{\mathbf{3}})]} \cdot e^{\mathcal{O}_e}\right) & \ln\left(e^{\mathbf{2}} \cdot \frac{\exp(\frac{\mathbf{1}}{\mathbf{3}}\boldsymbol{A})}{\exp(\mathbf{1}\boldsymbol{B})} \cdot e^{\mathcal{O}_p}\right) \\ \ln\left(e^{\mathbf{0}} \cdot \frac{\exp[\mathbf{C}(\mathbf{4C}+\mathbf{1})]}{\exp[\mathbf{D}\alpha^3]} \cdot e^{\mathcal{O}_\mu}\right) & \ln\left(e^{\mathbf{2}} \cdot \frac{\exp(\frac{\mathbf{2}}{\mathbf{3}}\boldsymbol{A})}{\exp(\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{B})} \cdot e^{\mathcal{O}_n}\right) \end{pmatrix}$$

[11] In asymptotic freedom, alike QED, the one-loop *beta*-function with n_f flavors in QCD $SU(3)_c$ is $\beta_1(g) = \frac{g^3}{(4\pi)^2}(-11 + \frac{2}{3}n_f)$ or $\beta_1(\alpha_s) = \frac{\alpha_s^2}{2\pi}\beta_0 = \frac{\alpha_s^2}{2\pi}(-11 + \frac{2}{3}n_f) = \frac{\alpha_s^2}{\pi}(\frac{-11N}{6} + \frac{1}{3}n_f)$, where $SU(3)$ give $N = 3$, $\alpha_s(Q^2) = \frac{g^2}{4\pi} = \frac{4\pi}{\beta_0 \ln[Q^2/\Lambda^2]}$ and g is a color charge, $\Lambda \approx 217$MeV is QCD scale. If $n_f < 16$, the coupling *is inversely proportional to* the energy scale. Because of asymptotic freedom, no free quark with the fractional charge is found in nature.

where $\mathbf{A} = \frac{1}{2\pi}\alpha^3$, $\mathbf{B} = \frac{1}{\pi^2}\alpha^2$, $\mathbf{C} = \frac{1}{2\pi}\alpha$ and $\mathbf{D} = \frac{3\pi}{\pi+1}$ for the Leptons and $\boldsymbol{A} = \frac{11}{9\pi}\alpha^{1/2}$, $\boldsymbol{B} = \frac{8}{\pi}\alpha^{1/2}$ for the Baryons. From (77), the g-factors of lepton and baryons are

$$\frac{1}{2}\begin{pmatrix} |g_e| & |g_p| \\ |g_\mu| & |g_n| \end{pmatrix} = \begin{pmatrix} \mathbf{1.0011596521807} & \mathbf{2.792847578} \\ \mathbf{1.0011659207972} & \mathbf{1.913042844} \end{pmatrix} = \tag{78}$$

$$\begin{pmatrix} \ln\left(\frac{\exp\left[\frac{\alpha^3}{2\pi}(\alpha^{-2}+\frac{2}{3})\right]}{\exp\left[(\frac{\alpha}{\pi})^2(\alpha^2+\frac{1}{3})\right]}e^{1+\mathcal{O}_e}\right) & \ln\left(\frac{\exp\left[\frac{1}{3}(\frac{11}{9\pi}\alpha^{1/2})\right]}{\exp\left[\frac{1}{\pi}(8\alpha^{1/2})\right]}e^{3+\mathcal{O}_p}\right) \\ \ln\left(\frac{\exp\left[\frac{\alpha}{2\pi}(\frac{2\alpha}{\pi}+1)\right]}{\exp\left[\frac{3\pi\alpha^3}{\pi+1}\right]}e^{1+\mathcal{O}_\mu}\right) & \ln\left(\frac{\exp\left[\frac{2}{3}(\frac{11}{9\pi}\alpha^{1/2})\right]}{\exp\left[\frac{1}{\pi}(4\alpha^{1/2})\right]}e^{2+\mathcal{O}_n}\right) \end{pmatrix}$$

12 Discussions

In this paper, we first checked the spin-statistics with the fine structure constant formula (**Table. 1**). According to Pauli's spin-statistics theorem, Bose-Einstein statistics are linked to mass and energy; Fermi-Dirac statistics are linked to the electric charge.

Table 1. The B-E and F-D statistics and fine structure constant

Statistics Type	Bose-Einstein	Fermi-Dirac
Number $n/2$	n=*even*	n=*odd*
Spin	0, 1, 2, $\cdots$	$\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\cdots$
Identity	Boson	Fermion
Particles	Photon, Gloun	Electron, Proton
Physical propertiy	Mass/Energy	Charge
Alpha	$\alpha^{-1/2}$	$\alpha^{1/2}$
Willis π	$\frac{\pi}{2} = [\frac{even}{odd}]^2$	$\frac{2}{\pi} = [\frac{odd}{even}]^2$
Logarithms	$\ln 10$	$\log e$
Golden Ratio	Φ	ϕ
Chemical potential	$-E$	$\pm E$
Related Field	Gravitational field	Electriomagnetic field

(a) Bose-Einstein statistics for the spin-integer boson are linked to *even* numbers. They always have negative chemical-potential "-" like the gravitational field, matching with the nucleon mass mainly contributed by the gluon vector boson;

(b) Fermi-Dirac statistics for the spin-half fermion are linked to *odd* numbers. They can have either negative or positive chemical-potential like the charged electric fields with "±" sign.

The fine structure constant formula (1) well interprets the Pauli spin-statistics theorem by simply flipping over $\alpha^{1/2} = \phi \log e / K\pi$ and $\alpha^{-1/2} = K\Phi\pi \ln 10$. 99% of nucleonic mass (proton and neutron) consists of gluon bosons in QCD, therefore, it is correct to say that the universal gravitational interaction is between the massive bosons. In contrast, the electromagnetic interaction is between the fermions.

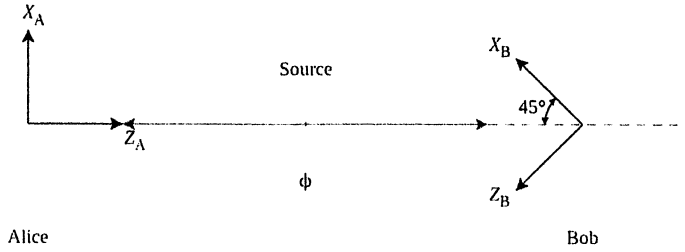

Fig. 18: The limited freedom spin states between Alice and Bob.

(6) shows that the electron spin state is due to pair electromagnetic interaction and must be locally chosen between $|s_+ = +\frac{1}{2}\rangle = |\uparrow\rangle = |0\rangle$ and $|s_- = -\frac{1}{2}\rangle = |\downarrow\rangle = |1\rangle$. It has significant importance to the quantum interpretation. (a) Since the incoming electron spins auto-opposite to the spin of the local occupied electron, it must randomly choose one of two possible states (as the Pauli Exclusion principle); (b) If there is an unoccupied state available, the incoming electron can choose its own spin state randomly (Hund's rule); (c) In any cases of (a) and (b), the choice must be between $|0\rangle$ and $|1\rangle$ (limited choice or half-relationship), Alice can not determine Bob 100%, but is still 50% related (**Fig. 18**), this is a limited freedom spin-state principle. Schrödinger's cat, [76] EPR paradox, [77] Copenhagen interpretation, [78] de Broglie-Bohm theory, [79]

Many-worlds uncertainty, [80] Bell's inequality, [81] the Von Neumann entropy quantum entanglement, [82] *etc*, are all debates on the same quantum effects ($|\psi\rangle \sim \pm 1/\sqrt{2}$) in different math-physical languages.

We also reviewed the developing history of S-matrix and QED anomaly calculations, proposed two new ways of calculating the anomaly (1) as a "*whole*" for leptons and (2) the new perturbative-logarithm formula for leptons and nucleons. This new perturbative a_e formula is a semi-theoretical equation, where c_2 and c_4 are derived by QED, while c_6 and c_8 are naturally obtained and finally explained in the electro-weak coupling. It is a simple and beautiful result; with the correct α value, Heisenberg's S-matrix becomes easy to understand. From the study of S-matrix, the Baryons g-factor used the $\alpha^{n+1/2}$ series to address the proton and neutron anomaly. Using the fine structure constant formula (1), the electro-weak model, Weinberg angle, CP violation, and topology of quark charge and confinement were also discussed.

We are exploring the dimensionless physical constants with the basic math constants, especially $\alpha^{1/2} = \log e^{\phi/K\pi}$ in (1), which is based on the beautiful identity $e^{\pm i\pi} + \Phi = \phi$, all in continued fractions

$$\begin{aligned}
\phi &= 0 + \tfrac{1}{1+}\tfrac{1}{1+}\tfrac{1}{1+}\tfrac{1}{1+}\tfrac{1}{1+}\cdots = 0.618033\ldots \\
\Phi &= 1 + \tfrac{1}{1+}\tfrac{1}{1+}\tfrac{1}{1+}\tfrac{1}{1+}\tfrac{1}{1+}\cdots = 1.618033\ldots \\
e &= 2 + \tfrac{2}{2+}\tfrac{3}{3+}\tfrac{4}{4+}\tfrac{5}{5+}\tfrac{6}{6+}\cdots = 2.718281\ldots \\
\pi &= 3 + \tfrac{1^2}{6+}\tfrac{3^2}{6+}\tfrac{5^2}{6+}\tfrac{7^2}{6+}\tfrac{9^2}{6+}\cdots = 3.141592\ldots
\end{aligned} \tag{79}$$

and $K = [1, \overline{6771}] = 1.000147$. [2,3,67]

$\phi - \Phi - e - \pi$ can also be presented as limitation formulas from

(a) Fibonacci numbers

The sequence F_n of Fibonacci numbers is defined by the recurrence relation with seed values $F_0 = 0$, $F_1 = 1$, $F_2 = 1$,

$$F_n = F_{n-1} + F_{n-2} \tag{80}$$

where the Fibonacci numbers $F_n \sim 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots.$

(b) Cloitre numbers[12]

Let's define $\{a_n\}$ and $\{b_n\}$ with $n = 1, 2, 3, 4, 5, \cdots$

$$a_{n+2} = a_{n+1} + \frac{1}{n} a_n \tag{81}$$
$$b_{n+2} = \frac{1}{n} b_{n+1} + b_n$$

with seed values $a_1 = b_1 = 0$ and $a_2 = b_2 = 1$, and the rest following "mirror images" in (81). (81) yields the Cloitre numbers $a_n \sim 0, 1, 1, \frac{3}{2}, \frac{11}{6}, \frac{53}{24}, \frac{927}{360}, \cdots$ and $b_n \sim 0, 1, 1, \frac{3}{2}, \frac{3}{2}, \frac{15}{8}, \frac{15}{8}, \cdots$.

The Fibonacci numbers in (80) and the Cloitre numbers in (81) give four limitations as

$$\begin{aligned} \phi &= \lim_{n\to\infty} \frac{F_n}{F_{n+1}} = 0.618033\ldots \\ \Phi &= \lim_{n\to\infty} \frac{F_{n+1}}{F_n} = 1.618033\ldots \\ e &= \lim_{n\to\infty} \frac{n}{a_n} = 2.718281\ldots \\ \pi &= \lim_{n\to\infty} \frac{2n}{b_n^2} = 3.141592\ldots \end{aligned} \tag{82}$$

With these math constants, we can calculate the charge, mass, g-factor and spin of the electron, explain the Dirac large number hypothesis, solve the mysteries of the FQHE, Quantum interpretations, lepton mass formulas, gravity mass information, *etc.* In fact, these extensive studies are also proving the correctness of the α formula (1). The fine structure constant formula (1) has a mathematical underpinning and is physically useful. The uniqueness of α is based on $6771 = 3 \times 37 \times 61$ from a double prime factorial equation that covers all the prime numbers. The unique number 6771 in math, matching with the unique information entropy ~137 in physics. We have found that the sphenic number

[12]http://www.pi314.net/eng/miroir.php

set {1, 3, 37, 61, 111, 183, 2257, 6771} also plays a critical role during the electron-positron pair production.

In nuclear physics, e^-e^+ pair production occurs when a high-energy gamma-ray photon interacts with the nucleus of an atom (Z protons). **Shockingly,** Bethe-Heitler mechanism shows that the magic number **183** = 3 × 61 (or **111** = 3 × 37) is involved in this process (**Fig. 19**). [73] The Bethe-Heitler mechanism has been experimentally confirmed, and this unexplained number 183 is well known in high energy particle physics. The Bethe-Heitler mechanism is also proposed for black-hole electron-positron pair production. [74]

(1) If $mc^2 \ll h\nu \ll 137\, mc^2 Z^{-\frac{1}{3}}$, then formula (22) (no screening) has to be integrated, giving

$$\Phi_{\text{pair}} = r_0^2 \frac{Z^2}{137}\left(\frac{28}{9}\log\frac{2h\nu}{mc^2} - \frac{218}{27}\right), \quad \text{(no screening } h\nu \gg mc^2\text{)}, \qquad (61)$$

a result which has been published in the preliminary note by Heitler and Sauter (*loc. cit.*).

(2) If $h\nu \gg 137\, mc^2 Z^{-\frac{1}{3}}$ (complete screening) we find

$$\Phi_{\text{pair}} = r_0^2 \frac{Z^2}{137}\left(\frac{28}{9}\log(183\, Z^{-\frac{1}{3}}) - \frac{2}{27}\right) \quad \text{(complete screening)} \qquad (62)$$

Fig. 19: The Bethe-Heitler formulas in 1934 [73]

The Bethe-Heitler formulas for the Integral Cross-section of e^-e^+ pair production are [73]

(a) In the case of an incoming gamma-ray photon $m_e c^2 \ll h\nu \ll 137mc^2Z^{-1/3}$,

$$\begin{aligned}\sigma_{e^+e^-} &= r_e^2\frac{Z^2}{137}\left[\frac{28}{9}\ln\frac{2h\nu}{mc^2} - \frac{218}{27}\right] \\ &= r_e^2\left(\frac{Z\log e}{K\Phi\pi}\right)^2\left[\left(3+\frac{1}{9}\right)\ln\frac{2h\nu}{mc^2} - \left(8+\frac{2}{27}\right)\right]\end{aligned} \qquad (83)$$

(b) In the case of an incoming gamma-ray photon $h\nu \gg 137m_e c^2 Z^{-1/3}$

$$\sigma_{e^+e^-} = r_e^2 \frac{Z^2}{137}[\frac{28}{9}\ln(\mathbf{183}Z^{-1/3}) - \frac{2}{27}] \quad (84)$$
$$= r_e^2(\frac{Z\log e}{K\Phi\pi})^2[(\mathbf{3}+\frac{\mathbf{1}}{\mathbf{9}})\ln(\mathbf{183}Z^{-1/3}) - \frac{\mathbf{2}}{\mathbf{27}}]$$

where $\mathbf{183} = 3 \times 61$ and $\mathbf{27} = 61 - 37 + 3$. Note that **3**, **8**, **1/9** and **2/27** also appear in the proton g-factor formula (73) and (74). It is not difficult to discern $2/27 = (1/3)(2/3)(1/3)$ and $1/9 = (1/3)^2$. The $1/3$ and $2/3$ in the above formulas are used many times for the g-factors of electron, muon, proton and neutron. Compare (83) with (73) and (74), both contain **3**, **8**, **1/3** and **2/27**. This proves the proton g-factor formula. Since the electron-positron creation is the starting point of fine structure constant application, this can be seen as a final proof of the fine structure constant formula (1), beyond any reasonable doubt. This important issue will be discussed in a separate paper.

As Einstein opined, "*In a reasonable theory there are no (dimensionless) numbers whose values are only empirically determinable.*" [59] "*Dimensionless constants in the laws of nature, which from the purely logical point of view can just as well have different values, should not exist. To me, with my 'trust in God' this appears to be evident, but there will be few who are of the same opinion.*" [60]

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